HEC PARIS

How to value a start-up?

The use of options to assess the value of equity in start-ups

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Executive summary

The question of start-up value is mostly interesting for two kinds of people: the entrepreneur, for whom it is a form of rewarding his work, and the outside investor, either as an individual or as a corporation for whom a fair value is necessary to profit from an investment in the firm.

The well-established techniques such as the DCF approach or the multiples valuation that exist for valuing mature companies are irrelevant to value start-ups. Why? Because contrary to mature profit-making firms, where activity can be reasonably forecasted from past results, this is impossible for new firms, where the actual market for a new product is completely unknown. They are also irrelevant to deal with the very binary kind of payoffs of an investment in a start-up: success or failure.

To truly assess those risks, but also the potential of value creation imbedded in start-ups, we need to recourse to more innovative methods, such as the real option method, which takes into account the potential of the firm to change its strategy through time and adapt to new market circumstances.

However, this complex method is not often used in practice, and is replaced by the venture capital method, which has flaws and has less theoretical justifications.

The problem is therefore to build a valuation technique that enables at the same time to understand the dynamics of the business, as the DCF approach enables to do, and to capture the risk profile of a start-up company, making it closer to the real option framework. This is what I have tried to do in the last part of this research paper.

But as start-up valuation mostly occurs in the framework of a capital increase, we will see that valuing a start-up cannot be limited to valuing the standalone value of the firm. Adjustments have to be made concerning the cost of capital and the shareholding structure, and here again, reasoning in terms of options gives precious insight.

Summary

Introduction

On the 7th of November 2013, Twitter was IPOed at a price of $26 per share, thus valuing it at $12.8 billion, namely 20 times its current turnover. By many standards in valuation, such a price can be considered as extremely high. But most surprisingly, on the first day of trading, the stock price increased by a staggering 73% to $45, implying a valuation of $30 billion, around 60 times Twitter's current turnover. And the price has hitherto remained at high valuation levels, around $45-$50.

However, when one looks at Twitter's financials, one realizes that the company is far from profitable yet, still incurring a $130 million loss for fiscal year 2013, and even the management does not believe it will be before 2015 at the very least. What would make Twitter so special that investors would be willing to buy a share of its equity for 60 years of the company's turnover given those assumptions?

The answer is simple: growth.

Indeed, just before it was introduced, Twitter revealed a 100% growth rate of its turnover in one quarter only. At such a pace, only three years and a half are needed to multiply the turnover by more than 100. In a business where most costs are fixed, one can easily understand how profitable such a business can become.

But growth sustainability is a big "if" for such a company which suffers from a decline in the price of its advertisement sales, and a higher competition from other technological firms whose revenues also stem from ads: Facebook, Google… Twitter's potential to reach profitability is therefore far from certain. It is not given that Twitter has already found the right business model to stay on business for a long time.

Twitter epitomizes many of the problems that arise in valuing a new business: high growth potential, very limited financial history, many financial indicators such as EBITDA or Net income are negative or very small, risk of failure is high.

All those factors make it very difficult to apply traditional valuation techniques in order to estimate the enterprise value.

Indeed, the discounted cash flow method (DCF) consists in measuring all the cash flows produced by the firm during all its lifetime and discount them at a rate that takes into account the risk inherent to the activity of the firm. Generally, three periods of time are considered: an explicit horizon during which cash flows can be forecasted quite accurately, a "soft landing", which correspond to the period when the firm gradually reaches maturity, and the terminal horizon, during which cash flows are assumed to grow at a constant low rate.

The matter with this method is that for a firm that has just been created or with low track record, it is extremely hard to know how long it will take the firm to reach maturity, to know how much of its potential market it will capture, or more pragmatically, it is hard to know if the firm will survive at all. In 100 new ventures, only 16% survive the first ten years[1]. Therefore, it is extremely hard to build a reliable business plan. Then, even if you suppose you are actually able to build a realistic business plan, how can you find the right discount rate? Indeed, the discount rate is usually computed on mature listed companies which bear less risk than a new venture that will most probably fail even before breaking even.

The other classical method consists in looking at other listed companies that are comparable (in terms of sector of activity, margins, growth rates…) and compare their value to their key financials: Turnover, EBITDA, EBIT, Net Income… Those multiples, applied to the company to be valued then easily yield the enterprise value or the equity value. But how to find a pertinent comparable when you try to evaluate a start-up, which is by definition an innovative company?

Now, why do we care about valuing start-ups? Or rather, who cares about start-ups value?

As explained above, one of the main features of start-ups is the uncertainty of the cash flows that are related to their activity. This makes debt financing extremely hard for them, for debt requires a steady stream of cash flows in order to be reimbursed. Besides, especially in activities with little asset base, bankers will be extremely cautious to lend because there will be no possibility to pledge some assets as a guarantee[2].

[1] *Vernimen, Finance d'entreprise* by Pascal Quiry and Yann Le Fur, 2014 edition p902. Based on a study between 2003 and 2013.
[2] In practice, the entrepreneur can borrow money by pledging some of his own assets (house, car…) but then this debt is not part of the firm's debt.

This impossibility, or at least weak probability to be able to finance by debt makes equity financing an almost unavoidable fact. And that's where the importance of the firm value comes into play.

The first people to take part in the equity financing of the firm is the entrepreneur, or a group of entrepreneurs, pooling together their resources to finance their new venture. At this stage, what matters is that everyone gets a share of equity that corresponds to the value he will bring to the future company.

As those resources are generally not sufficient to start a proper business, they also address their "fools, friends and family" to get a little extra help against a symbolic share of equity. This money being invested not for financial reasons but mostly for the personal attachment to the people creating the company, it has been nicknamed "Love Money".

However, at one point or another, depending on the nature of the business, the firm will have to look for outside investors, who will intervene at different steps of the company life: business angels, venture capitalists and private equity funds. To them, what matters is to pay a price for the shares that will enable them to make a good capital gain[3]. Therefore, if the equity value of the firm is very high when they become shareholders, and that the firm's results are not good, it is very unlikely that they will be able to renegotiate their shares at a good price, and they will certainly lose money. On the other hand, if they enter at a very low price, the entrepreneur will be largely diluted, and might, in the end, be unmotivated so the company withers and its shares finally become worthless.

All those investors do not get the same conditions when they enter the company's equity. At each round of financing, the new investors will negotiate specific contracts with the previous shareholders in order to protect their interests, such as ratchet, veto right, drag-along or tag-along clauses. This makes that the structure of equity in a start-up is very heterogeneous. The value of equity for each investor has to take into account those various features in order to determine the firm's value for each investor.

Finally, the last point in time when start-ups value matters is when this company wants to become public. An IPO is nothing else than selling the shares of the company to the public. Fixing a very high price for the IPO as Twitter in our initial example enables former investors

[3] It might be less true for business angels who can from time to time invest in a business because they like the idea or to benefit from tax rebates, rather than getting a capital gain, but there is at least a desire to be able to get one's capital back.

to cash out and get their capital gains, but might in the end be a disaster for public investors if Twitter doesn't deliver the goods.

Therefore, how to take into account start-ups specific features in order to value them?

The traditional DCF and multiple methods are adapted for mature businesses but limited for start-ups valuation purposes. To better capture start-ups specific features, it is necessary to use more sophisticated approaches, such as the real options method which enables to capture the inherent flexibility of the investment in a start-up, or a more pragmatic one as the methods developed by venture capitalists. After seeing the limits of those approaches, we will try to develop a new one inspired from real options, and we will also see how to take into account the specificities of start-ups capital structure to adjust the value for the various actors.

I) The limitations of the traditional valuation approaches in valuing a start-up

Well-established methods exist for standalone valuation of mature companies. However, those methods prove to be quite irrelevant when applied to start-ups.

1) The DCF approach

a) Justification and principles

The justification of the Discounted Cash Flow method is very simple in theory. As Aswath Damodaran, one of the world's leading experts on valuation puts it, "The intrinsic value of a cash-flow generating asset is a function of how long you expect it to generate cash-flows, as well as how large and how predictable these cash-flows are"[4].

One of the basic rules in finance is that time is money: any rational investor will prefer to have one unit of cash today rather than one unit of cash tomorrow. But unless this investor is also risk neutral, he will also prefer a project that will yield one unit of cash tomorrow for sure rather than one yielding zero or two with equal probability. To take that into account, the provisional cash flows of any project are discounted at a rate engulfing both time and risk; for any cash flow at time t, its value right now is:

[4] *The Dark Side of Valuation, Valuing young, distressed and complex businesses*, Aswath Damodaran,2nd edition.

$$\mathrm{PV}(\mathrm{CF_t}) = \frac{\mathrm{CF_t}}{(1+\mathrm{r})^{\mathrm{t}}}$$

Where r is the discount rate. We can remark that the higher the r, the lower the present value. Hence a higher discount rate signals a higher risk linked with that project.

By extension, the value of any project is therefore the sum of all discounted cash flows during its life:

$$\mathrm{PV_{project}} = \sum_{\mathrm{k}=1}^{+\infty} \frac{\mathrm{CF_k}}{(1+\mathrm{r})^{\mathrm{k}}}$$

Therefore, to come back to Damodaran's explanation, we do observe from this formula that the value of a project indeed depends on:

- How long do you expect it to generate cash-flows
- The size of those cash-flows
- The risk (or predictability) of those cash flows.

For a company which already has a stable activity, in order to have a reasonable idea of the cash flows that will occur in the next three to five coming years, we have to build a business plan. This period is called the "explicit horizon". To do so, we can first observe the firm's past performances, and, by discussing with the management, looking at the market in which the firm evolves, at its strategy, have a rough idea of how sales, margins and cash generation will evolve. It is very unlikely on a mature market that those elements will change significantly in the short run.

Then we estimate how the firm will generate cash flows at maturity. Obviously, it is very unlikely that it will be able to have fast growing cash flows forever, otherwise this company would become the only one in the economy.

Between the two, a "soft landing" is assumed, in which growth, margins, investments progressively decline to attain their terminal value.

All cash flows are then discounted using the company's weighted average cost of capital or WACC. As its name implies, the WACC is a weighted average of the cost of equity

K_E (required rate of return for the company's shareholders) and the market cost of debt K_D after tax (how much would it cost to refinance the company's debt).

$$WACC = K_E \times \frac{V_E}{V_E + V_D} + K_D \times (1 - \tau) \times \frac{V_E}{V_E + V_D}$$

Where K_E is the cost of equity, V_E the market value of the company's equity, V_D is the market value of debt and τ is the marginal tax rate.

The cost of equity can itself be calculated using the Capital Asset Pricing Model developed by Sharpe[5], Lintner[6] and Markowitz. This model shows that for any investor able to fully diversify his portfolio, the required rate of return on a given stock should only be the one of the non-diversifiable risk linked with his portfolio. Said in another way, it will be equal to the return of a risk-free investment plus a premium linked to the stock and the non-diversifiable risk:

$$K_E = R_F + \beta \times (R_M - R_F)$$

Where R_F is the risk-free rate, R_M the weighted average of all securities returns on the market and β is the correlation between the security and the market returns.

As Modigliani and Miller showed[7], the WACC should depend only on the risk of the company's activity, and not on the proportion between debt and equity in the company's financing.

The sum of the discounted cash flows on these three periods gives the enterprise value, and when net debt is subtracted, one gets the equity value. The method is as simple as that.

b) Limits in the case of start-ups

Why wouldn't this method work for start-ups?

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[5] Sharpe, William F. (1964). "Capital asset prices: A theory of market equilibrium under conditions of risk", Journal of Finance, 19 (3), p 425–442

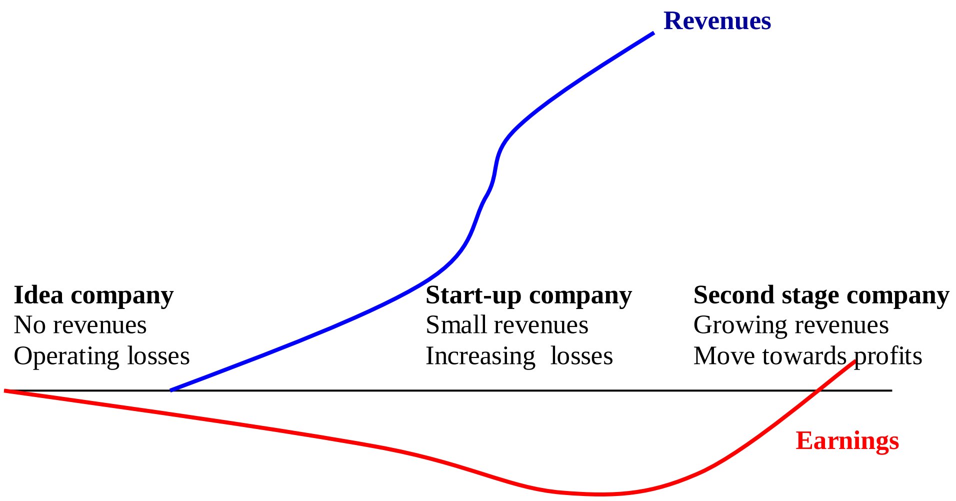
[6] Lintner, John (1965). "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets", Review of Economics and Statistics, 47 (1), 13–37.

[7] Modigliani, F.; Miller, M. (1958). "The Cost of Capital, Corporation Finance and the Theory of Investment". American Economic Review 48 (3): 261–297

First because this method already presents some flaws for mature companies, that are only exacerbated when trying to apply it to a start-up.

Chief among all, there is the reliability of the business plan. All cash flows being computed from this source, it has a key impact on the firm's value. But as we have said, for a stable company, those figures can be estimated from past results. We generally do not have such history for a start-up, which is by definition young, and even if we do have it, it won't be representative of the firm's future earnings.

Indeed, at the beginning of their lives, start-ups incur losses because their revenues are not large enough to compensate for their fixed costs. Even when they start generating revenues, their losses increase because they have to invest to cover a larger demand without having enough volumes to compensate for it. It is only when they manage to generate enough sales that they are able to compensate for their expenses and break even[8]. Therefore, we cannot use start-ups past performances to forecast their future ones.



But Damodaran points out an even more interesting point on that subject. In reality, the DCF method evaluates cash flows from existing assets but also the "expected growth from both new investments and improved efficiency on existing assets". The matter for start-ups is that their existing assets are very small, and because of the lack in financial data, it is hard to estimate the real performance of those assets. Therefore, most of the value will come from "growth assets", namely the assets that the firm will put into place with future investments, but we have no indication about their profitability.

[8] The graph is taken from Damodaran's *Dark Side of Valuation*, p 214.

Another flaw of the DCF method for standard companies is that most of the company's value is embedded in its terminal value, i.e. in the cash generated when the company's has reached maturity. But for a start-up, this terminal value would easily represent 90 or even 100% of the firm's value. Besides, one can have no idea of the time when the company reaches maturity, and what it will be like when it does.

Second, we have seen that another factor playing a huge role in the DCF method is the discount factor. Traditionally to estimate this discount factor, one looks at comparable listed companies in the same sector with similar growth rates and margins and computes their unlevered β to apply it to the financial structure of the company to be valued.

The matter is that it would be very difficult to find a comparable for a start-up for many reasons: a listed company will clearly not have such a quick growth as a starting one, the activity will certainly differ as a start-up is an innovative company with a product or service that has yet to be tested. All those reasons render very difficult the application of the DCF method to start-ups.

c) Possible adjustments to the case of start-ups

However, according to Damodaran, it is possible to make a certain number of adjustments to adapt the DCF approach to start-ups: "There is no reasons why young companies cannot be valued systematically"[9]. To do so, one must combine a top-down and a bottom-up approach in order to build a consistent business plan for the start-up and therefore be able to compute its provisional cash-flows.

The first step, which is the basics of any start-up business plan, is to define the potential market, and the potential share that the company will take on that market. This depends on the capacity of the management, but also on the company's investments to get this share. Then we have to estimate margins; to do so, we look at other steady companies in the same sector, to get a "target margin", and we define a "pathway to profitability", in which the firm will gradually reach those target margins. One has to check that the investments are consistent with the growth rate which is assumed. In this approach, as investments and operating income are estimated separately is that both numbers might be insufficient. Their

[9] The Dark side of valuation, p 225

consistency should therefore be checked by computing the implied return of capital and its evolution through time:

$$Implied\ Return\ on\ Capital = \frac{Expected\ Operating\ Income\ after\ tax_t}{Capital\ invested\ in\ firm_{t-1}}$$

This return should necessarily tend to the sector's average return on capital. If the return is significantly higher, it means that investments are insufficient and conversely.

The second step is the bottom-up approach, which starts by analyzing the firm's investment capacity to deduct the market it can reach. Depending on the number of units sold, we deduce the operating costs and taxes paid. The last step is to deduce if extra investments will have to be made in order to preserve the company's growth capability, in particular working capital requirements if large growth has to be sustained.

"As a general rule, bottom-up approach of cash-flows yield lower expected cash-flows and earnings, because we work with capacity constraints. Consequently, these approaches are more suited for businesses that either face significant restrictions on raising additional capital (…) or are dependent on a key person or people for their success. As a general rule, personal-services businesses (medical practices, a plumbing business, a restaurant) are better valued using this approach rather than the top-down approach, unless the service can be franchised or replicated easily"[10].

To try to prove his point, Damodaran took in another article the example of Linkedin[11] when it was IPOed, and provided his own way of valuing the company. Linkedin is a very good example of a start-up with limited history, negative cash flows, fast growth… His reasoning is as follows:

- Given the size of the advertising business and the manpower that Linkedin currently has, it should have a growth of 50% a year in revenues for the next five years, starting from $243m, and then scale down linearly to a 3% perpetual growth rate at year 10.

- The company currently has operating margins of $20m (c. 8% of revenues), and those margins should increase linearly to the industry average, 15%. EBIT after tax is 12m (tax rate of 40%).

[10] The Dark side of Valuation, p. 234.
[11] "Valuing Growth companies" AAII Journal December 2011 p.30

- The firm has to reinvest one dollar in capital to get $2.14 of additional revenues (median across firms in the sector).

- The cost of capital will be first assumed to be 12% (risk-free rate is 3%, beta is 1.8, market risk premium is 5%), and will linearly decrease from year 5 to reach the industry's average of 7.5%. Its ROCE will stabilize at 10% forever.

- He chooses not to adjust for survival because the firm does not depend on key personnel, has the first mover advantage in a sector in which it is key (social networks), and has no debt (and should remain so after its IPO).

The first step, computing the free cash-flows to the firm for the first 10 years is then straightforward (assumptions are in bold):

Year	Revenues	Growth rate	Margin (%)	EBIT	EBIT after tax	Investment	FCF	Cost of capital	Discount factor	DCF
0	**243**	**50%**	8,2%	**20**	12			**12%**		
1	365	**50%**	8,9%	32	20	57	-37	**12%**	89,3%	-33
2	547	**50%**	9,6%	52	32	85	-53	**12%**	79,7%	-42
3	820	**50%**	10,3%	84	52	128	-76	**12%**	71%	-54
4	1230	**50%**	10,9%	135	83	192	-109	**12%**	64%	-69
5	1845	**50%**	11,6%	214	133	287	-154	**12%**	57%	-88
6	2594	41%	12,3%	319	198	350	-152	11,1%	51,1%	-78
7	3404	31%	13,0%	441	274	378	-104	10,2%	46,3%	-48
8	4146	22%	13,6%	566	351	347	4	9,3%	42,4%	2
9	4660	12%	14,3%	667	414	240	174	8,4%	39,1%	68
10	4800	**3%**	**15%**	720	446	65	381	**7,5%**	36,4%	139
									Sum	**-203**

And then computes the terminal value with the formula:

$$TV = \frac{EBIT(1-\tau) \times (1+g) \times [1 - \left(\frac{g}{ROCE}\right)]}{k-g}$$

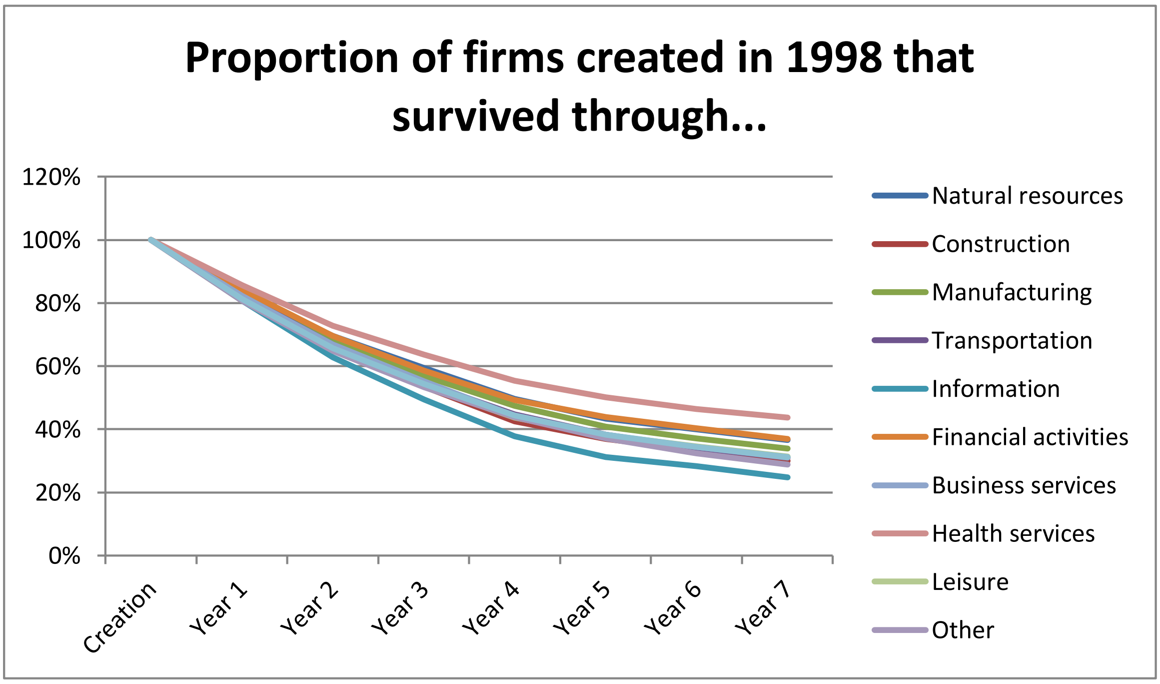
Where τ is the tax rate, g the perpetuity growth rate, k the cost of capital. The term [1-(g/ROCE)] is here to take into account reinvestment. This yields $7,152m[12], which is $2,603m in present value. The value of the company is therefore $2,400m to which he adds the net cash balance, $93m, yielding an equity value of $2,494m. After adjusting for options and voting rights, he finally sets the value of the shares to be offered at a price of $20.94 per share, equivalent to a $1,980m equity value.

[12] If computed without any roundings

However, the company was IPOed at an opening price of $43 per share and from the first day, the price went up to $100. His value is therefore far away from the market.

He argues: “If you follow these rules, will you be rewarded? Unfortunately, I cannot make that promise. In fact, it is entirely possible that you will watch neighbors and friends who use far less sensible approaches to investing make money on these stocks as they rise, while your portfolio languishes. It is also likely that you will watch these same people lose that money just as quickly when the “correction” comes (as it inevitably will)”. Well, Linkedin share price is today a $193, and the “correction has not come yet”; either the market has been foolish for three years or Mr. Damodaran has just been wrong.

I do believe that such an approach enables to better understand the business in terms of potential, but what is this potential worth if the firm doesn’t make it to maturity? This is more than likely given the mortality rate in start-ups[13]:



Therefore, this doesn’t tackle the question of risk, and by extension, the question of the discount factor, which plays an essential role in the DCF approach. In the example of Linkedin for instance, a 12% discount rate for a firm with negative cash flows seems way too low! Besides, Damodaran himself uses words such as “management capacity”, “pathway to profitability”, which rely too much on the analyst’s judgment. This cannot solve the problem of discrepancies between the estimations of the owner of the firm, who will necessarily be

[13] The Dark side of Valuation, p. 216

optimistic about his business and the opinion of an external analyst who doesn't necessarily have a good view of the market or of the management's capacity.

This vision is summed up by Dermot Berkery: "None of the traditional valuation techniques – discounted cash flows analysis, payback formula and so on – seem to be relevant. Everyone involved recognizes that these highly analytical approaches require so many assumptions that they become meaningless – there are too many moving variables. Also, the outcomes for many variables can be binary: success or failure. A market can take off, or might not. A product might reach a particular specification, or might not. Regulators might approve the product or they might not"[14].

But now that we have tackled the question of the reliability of the business plan, couldn't we use the multiples approach, which doesn't rely on the discount rate, to value a start-up?

2) The multiple approach

a) Principles of the method

The principle of the multiples method is even simpler than the discounted cash flows method: the best way to know the price of a company is to know how similar companies are priced. This price has to be scaled to a certain indicator of profit generation, such as sales, EBIT, EBITDA or earnings, the same way as in real estate, where the number of squared meters is taken as a reference to get the value of a flat.

Those multiples can apply either to the enterprise value (equity value plus net debt) or only to the value of equity. Sales, EBIT and EBITDA multiples enable to compute the enterprise value, while the price earnings ratio (P/E ratio) enables to give the equity value.

For example, let's assume that we want to evaluate company A, which is comparable to the listed company B, and that B is listed on the stock market.

[14] Dermot Berkery, *Raising Capital For The Serious Entrepreneur*, p 141

All figures in €M	**A**	**B**
Revenues	100	750
EBITDA	15	80
EBIT	9	60
Net income	5	40
Net debt	20	74
Market capitalization	N/A	672

If we use the EBITDA multiple, we can see that B enterprise value (Market Cap + Net debt) is equal to 9.33 times its EBITDA: (672+74)/80≃9.33. We therefore apply this multiple to A's EBITDA to get A enterprise value: A_{EV}=9.33x15≃140. Then we subtract net debt to get A's equity value: A_{EQ}=140-20=120. The same principle could be used with the revenues or EBIT multiple.

If we use net income as a multiple (often called the P/E ratio, price earnings ratio), we first get A equity value and then induce its enterprise value. P/E_B=672/40=16.8. Therefore A_{EQ}=16.8x5=84. Then A_{EV}=84+20=104.

b) Limitations

This method has to be simple, but depending on the multiple which is used, several points should be checked before starting to consider another listed or recently traded company as a comparable. The following table gives a summary of the different points that have to be raised:

Multiple	**Conditions of use**
Revenues	Same growth rate, same margins, same risk, same tax rate, same investment policy
EBITDA	Same growth rate, same risk, same tax rate, same investment policy
EBIT	Same growth rate, same risk, same tax rate
P/E	Same growth rate, same risk, same financial structure, positive earnings
PEG	Not really pertinent
Industry specific	Limited to very similar firms in the same sector

That's where all the problems with using the multiples method to value start-ups start to appear. As we can see above, having the same growth rate is a key condition to be able to compare two companies, but it is very unlikely that a listed company will have the same growth rate as a starting business. As a consequence, it is also very unlikely that their investment policy will be similar.

Being exposed to the same kind of market risk is also a key factor, which is very hard to be met, because a listed company will have a stable and well identified market (Twitter might be one of the few exceptions), whereas a start-up has no real idea of its market.

The similarity of the financial structure is also problematic when it comes to use the P/E ratio. Indeed, as we have stated in introduction, start-ups are almost entirely financed by equity[15], whereas listed companies, because they have more foreseeable cash flows, can have some debt in their balance sheet. This problem is however less systematic.

A second matter comes with the multiples themselves: in a start-up, EBIT, EBITDA and net income are most likely to be negative[16], so the valuation implied by the multiples would be meaningless. As for the revenue, it might be positive, but so low that the valuation would not often meet the book value of equity. In practice, it is possible to rectify this problem by estimating those figures three to four years from now, as we will see in the venture capital method, but to do so we are confronted to the same problem of the business plan reliability as with the DCF method.

This point drew many venture capital funds and entrepreneurs at the beginning of the internet bubble of 2000 to use other multiples, such as the number of users on a website, to infer the value of the company. However, as those multiples are not linked with profit generation, they led to absurd valuations that appeared cruelly when the bubble burst.

Damodaran stresses another very interesting limitation of this approach in the case of start-ups, more specifically when multiples are extracted from past transactions. Let's assume we want to value a start-up named A, and that we have the chance to find a good comparable company, B, which is also a start-up and has just finished a capital increase. We have all the

[15] Start-ups also use convertible bonds for funding, but in terms of valuation, those should be considered as equity because of the risk there is that they can be converted into shares of the company.
[16] See graph page 11

data necessary to compute B’s multiples. In theory, we should be able to infer the value of A. However, this is not the case, Damodaran explains, because we have no clue about the contractual clauses of the shareholder agreement, and such clauses because they offer preferential claims on equity (e.g. ratchet clauses) affect the value of equity. We will see in the last part that we can value the impact of those preferential claims on the value of equity, but if we don’t have access to the shareholder agreement, which is often the case, we won’t be able to infer the right value of the company.

According to him, besides the further adjustments that have been listed, we should take into account the probability that the firm survives over the estimation horizon, given that the multiples analysis is based on a going concern hypothesis. The discount rate that we use to discount the future value of the company has to take into account the fact that this company is not diversified.

II) Real option valuation as well as venture capitalist methods give a better view of a start-up value

In practice, the previous methods fail to provide a good view on start-ups valuation because they either focus on one business plan which is very unlikely to happen in reality or try to infer the value of a start-up by comparing it with other companies that have in fact little in common maybe except the sector of activity. Contrary to mature companies, start-ups can evolve in many ways, which can’t be grasped by the methods presented above; the real option method is a very good way to summarize it. We will also look at the methods used by practitioners to value start-ups, and see if they indeed enable to get a more realistic value.

1) Real options capture two essential dimensions of a start-up: time and adaptability

a) The creation of a start-up is an adaptive and step-by-step process

The very process of creating a start-up is a trial and errors process in which the entrepreneur constantly adapts his project to the results he obtained in order to better grasp the market.

This concept is very well developed in Eric Ries's *The Lean Start-up*, where he explains that a start-up is a continuous process of finding what the right product for the targeted market is. To do so, the company is constantly in a "Boucle-Build-Learn" process: depending on its results, the company will build new very simple products just to validate the visions it has on the market. At the end of this process, it can decide to accelerate in the same direction, to abandon the product, or to "pivot", meaning a complete shift in the features of the product sold. Through this constant path of innovation, the company will progressively have a clear idea of the right product for the market.

This step-by-step approach is key in order to understand the way start-ups have to be funded. Each round of financing should enable the company to get from one step to the other.

Let's imagine a start-up that would like to create a new drug: the first step would be to make clinical research in order to obtain a functional drug, then it would have to get approval for tests and, if those are successful, to get approval for the commercialization, and then sell the product.

Because of the very weak probability of a new drug to make it from R&D to commercialization (less than 3%[17]), it would be extremely risky to fund this project so that it can make it to the end. If the project is fully funded but that research fails to obtain an operational medicine, what would be done with the remaining funds? There is a very high probability that they will be misused to keep searching for a product that might not work in the end.

On the contrary, once a step has been reached, there is a much higher probability to get to the next one[18]:

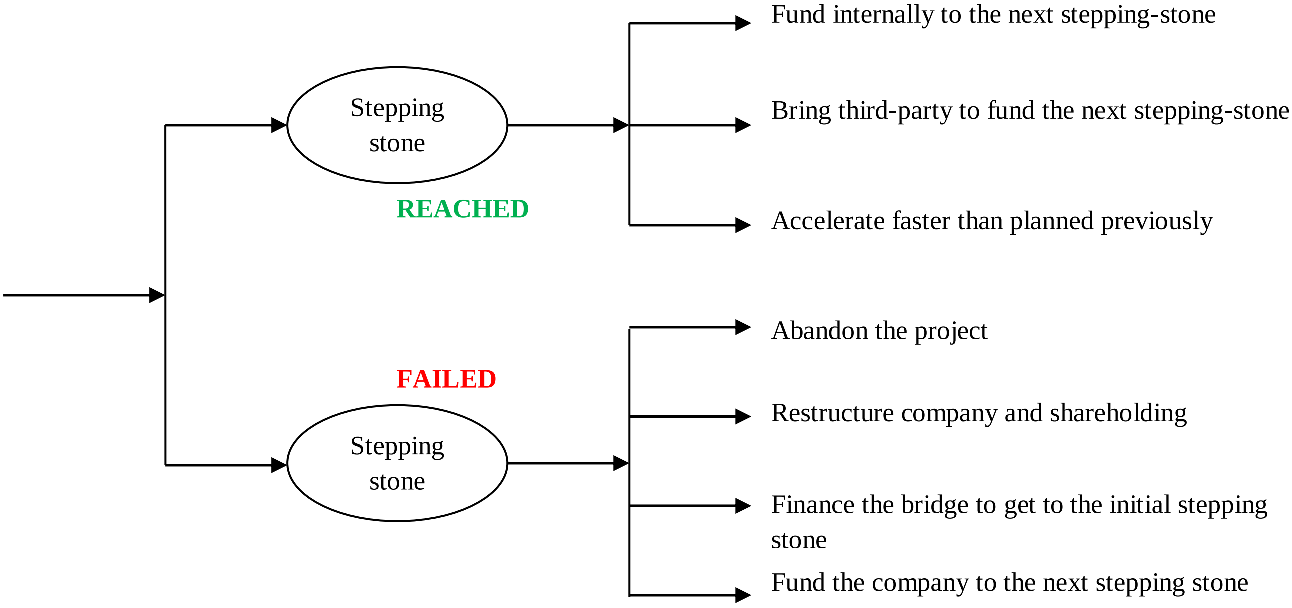
Phase	**Probability of transition**
R&D to preclinical	25%
Preclinical to Phase I	45%
Phase I to Phase II	60%
Phase II to Phase III	65%
Phase III to registration	75%
Registration to market approval	85%

[17] Source AFIC 2007, quoted by Jean Rédis in *Finance entrepreneuriale*
[18] Ibid.

Therefore, it is a much better strategy to fund the project only from R&D to preclinical tests at the beginning: if it succeeds, then we can fund the project for the next phase, if not we can stop there or try to get extra funds to reach this first step.

Even though the figures and steps presented above are specific to the drug discovery industry, the process remains true for any kind of business: it will have to go through a design of the product or service, then through its creation, and afterwards to its commercialization. Some other steps might appear depending on the sector (e.g. authorities approval for regulated businesses) but it doesn't change this principle of funding the process step-by-step.

This funding is process is largely developed by Dermot Berkery[19], which he calls "the stepping stones method". He explains that the entrepreneur should identify different steps for his project, and only find enough funds to get to the next stepping stone. Depending on the result, he will then be able to readapt his strategy according to the following scheme:



We can now see why the traditional methods presented before don't allow to estimate the value of a start-up correctly: the DCF approached is based only on one central business plan which is unlikely to happen, given the variety of ways a start-up can evolve. This is showed by Brennan and Schwartz in their article "Evaluating natural resources investments"[20]. Same way, in the comparable method, even if the start-up is comparable today

[19] From Dermot Berkery, *Raising venture capital for the serious entrepreneur* p39
[20] *Journal of business* 1985

to a listed company, there is a high probability that its margins, growth rate and other indicators will vary depending on the strategy moves made at each steps.

b) Principles of the real option method

To evaluate a start-up, we therefore need some tools that are able to grasp this complexity and flexibility, and this is one of the solutions brought forward by the real option method. It consists in valuing the flexibility inherent to a project and adds it to the classical calculation of the NPV.

"A real option is a right and not an obligation to take a decision linked with an investment in a physical asset at a given date (European option) or during a given period (American option) for a given price known in advance, called the strike price. The option is said to be real, as opposed to classic options (financial options) because the underlying is not financial (…). The decision to exercise the real option depends on the materialization of a non-foreseeable event (…)"[21].

Any investment opportunity can therefore be seen as a call option, in the sense that the firm has the right and not the obligation to invest. If this investment fails, the firm will never lose more than its investment thanks to the limited liability principle, the same way that when you buy a call option, you can never lose more than the premium paid at the beginning if the option is finally out of the money at maturity. But if things go right, you will be able to benefit from the upside procured by the investment.

On this basis, Amran and Kulatilaka[22] identify seven types of real options, which will enable to understand what they really represent:

- Growth option: a firm can make an investment today, but next year, if the results are good, it will be able to further invest to increase its revenues.
- Waiting option: obtaining a license for a business enables to launch the business at any moment during the validity of the license.
- Sequential development option: as in the previous example for the drug development, we see that each stepping stone enables to get further or to stop the project.

[21] From *Options Réelles* by Olivier Levyne and Jean-Michel Sahut
[22] "Real options: managing strategic investment in an uncertain world", Oxford University Press 1998

- Option to abandon: you open a new restaurant; if the business doesn't work well enough you can sell back the restaurant to another person.

- Exchange option: you have the choice to use petrol or electricity as a primary source of energy for your plant.

- Extension or reduction option: when you buy a plant, you can build an extension of the factory if you think that the capacity of production have to be increased.

- Learning option: you can wait to implement your project until new information arrives.

c) Valuation of real options and application to the start-up case

Such examples enable us to draw a parallel[23] between the components of a financial and a real option that will be helpful to determine the price of those options, and therefore answer to the question of their value in the framework of start-ups.

Real option	Financial option	Notation
NPV of the assets to be acquired for the project	Price of the underlying	S
Investment to make the project	Strike price	E or K
Possible delay of the decision	Time to maturity	T
Time value of money	Risk-free rate	R
Risk measure of the project	Volatility of the underlying	σ

Once we know those elements, we can then apply the Black-Scholes formula or a Cox-Rubinstein model to get the value of the option.

Taking back the previous notations, the Black-Scholes formula states that for a project requiring an initial investment of K, with an NPV of S, maturing in τ years, with volatility σ and given the risk-free rate of r, the value of the real option would be:

$$\boxed{C = S.\Phi(d_+) - Ke^{-r\tau}.\Phi(d_-)}$$

Where Φ is the cumulative distribution function of the standard Gaussian variable, and

[23] From *Options Réelles, intégrer risque et flexibilité dans les choix d'investissement*, Levynne et Sahut p 36

$$d_+ = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

And

$$d_- = \frac{\ln\left(\frac{S}{K}\right) - \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} = d_+ - \sigma\sqrt{\tau}$$

Technical note: demonstration of the Black-Scholes formula[24]

Let's assume we have one risk-free asset yielding the risk-free rate and one risky asset which returns follow the following dynamic (called a geometric Brownian motion):

$$\frac{dS_t}{S_t} = rdt + \sigma dB_t$$

(t denotes time and B is a Brownian motion under Q, the risk-neutral measure).

Then under the no arbitrage condition, the price P(G) of the call option is

$P(G) = E^Q(e^{-r\tau}G)$

$= E^Q(e^{-r\tau}\max(S_\tau - K; 0))$

$= E^Q\left(e^{-r\tau}S_\tau \mathbb{1}_{(S_\tau > K)}\right) - E^Q\left(e^{-r\tau}K\mathbb{1}_{(S_\tau > K)}\right)$

$= \underbrace{E^Q\left(e^{-r\tau}S_\tau \mathbb{1}_{(S_\tau > K)}\right)}_{A} - \underbrace{E^Q\left(e^{-r\tau}K\mathbb{1}_{(S_\tau > K)}\right)}_{B}$

We have

$A = E^Q\left(e^{-r\tau}S_\tau \mathbb{1}_{(S_\tau > K)}\right)$

$= E^Q\left(e^{-r\tau}S_0 e^{r\tau - \frac{1}{2}\int_0^\tau \sigma^2 dt + \int_0^\tau \sigma dB_t}\mathbb{1}_{(S_\tau > K)}\right)$

$= E^Q\left(S_0 e^{-\frac{1}{2}\int_0^\tau \sigma^2 dt + \int_0^\tau \sigma dB_t}\mathbb{1}_{(S_\tau > K)}\right)$

Let's introduce a new probability measure Q'. We have $\frac{dQ\prime}{dQ} = e^{-\frac{1}{2}\int_0^\tau \sigma^2 dt + \int_0^\tau \sigma dB_t}$

[24] From Black and Scholes "The pricing of options and corporate liabilities", Journal of political economy 1973 and Nizar Touzi "Advanced asset pricing, risk-neutral valuation in continuous time".

Then $A = S_0 E^{Q'}(\mathbb{1}_{(S_\tau > K)})$

$A = S_0 Q'(\mathbb{1}_{(S_\tau > K)})$

If we set $dB'_t = dB_t - \sigma dt$, then by Girsanov's theorem, B'_t is a Brownian motion under the new probability measure Q'.

Hence $\ln(S_\tau) = \ln(S_0) + \int_0^\tau \left(r - \frac{\sigma^2}{2}\right) dt + \int_0^\tau \sigma(dB'_t + \sigma dt)$

So $\ln(S_\tau) \underset{Q'}{\sim} \mathcal{N}\left(\ln(S_0) + \int_0^\tau \left(r + \frac{\sigma^2}{2}\right) dt\,; \int_0^\tau \sigma^2 dt\right)$

After centering and normalizing, we therefore have

$$\boxed{A = S_0 \phi\left(\frac{\ln\left(\frac{S_0}{K}\right)}{\sqrt{\int_0^\tau \sigma^2 dt}} + \frac{1}{2}\sqrt{\int_0^\tau \sigma^2 dt}\right) = S_0 \phi(d_+)}$$

And if r and σ are constant with respect to t (which was assumed at first), the integrals can be simplified and we get:

$$\boxed{A = S_0 \phi\left(\frac{\ln\left(\frac{S_0}{K}\right)}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau}\right) = S_0 \phi(d_+)}$$

Same way,

$B = K e^{-r\tau} E^{Q}\left(\mathbb{1}_{(S_\tau > K)}\right)$

$= K e^{-r\tau} Q\left((S_\tau > K)\right)$

By applying Ito's formula to the process S, we get

$$d(\ln(S_\tau)) = \frac{1}{S_\tau} dS_\tau - \frac{1}{2}\sigma^2 dB_t$$

$$= \left(r - \frac{\sigma^2}{2}\right) dt - \frac{1}{2}\sigma^2 dB_t$$

Which yields, by integration (assuming again σ is a constant of time):

$$\ln(S_\tau) = \ln(S_0) + \left(r - \frac{\sigma^2}{2}\right)\tau - \frac{1}{2}\sigma^2 B_t$$

Hence,

$$\ln(S_\tau) \underbrace{\sim}_{Q} \mathcal{N}\left(\ln(S_0) + \left(r - \frac{\sigma^2}{2}\right)\tau; \sigma^2\tau\right)$$

Which gives after centering and normalizing

$$B = Ke^{-r\tau}Q\big((S_\tau > K)\big)$$

$$\boxed{B = Ke^{-r\tau}\phi\left(\frac{\ln\left(\frac{S_0}{K}\right)}{\sigma\sqrt{\tau}} - \frac{1}{2}\sigma\sqrt{\tau}\right) = Ke^{-r\tau}\phi(d_-)}$$

Hence the Black-Scholes formula.

Let's see how this principle could work on a start-up. We come back to our example of a new venture for a new drug. Given the risk of the project, the DCF value of the project is 800 while it would require an investment of 1000. Theoretically, this project should not be overtaken. However, if after the five-year development period, the drug does make it through, the company will be able to reach not only the domestic market, but also foreign ones. Doing so would require an investment of 150, and would generate 50% of additional cash-flows.

Now, to be able to use the Black-Scholes method, the hardest step is to calculate the volatility of the project. This can be done in three ways:

- Compute the historical variance of this firm's projects (impossible to do for a start-up)
- Make several scenarios and look at the volatility of cash-flows across cases (Monte-Carlo method)
- Compute the variance of projects for firms in the same industry

Here we will assume that we have a volatility of 30%, obtained from a comparison with other pharmaceutical companies. Our parameters are as follow:

Parameter	**Notation**	**Value**
NPV of the project	NPV	800
NPV of expansion cash-flows	S	400 (=800x50%)
Cost of investment	K	150
Time to expiration in years	τ	5
Risk-free rate (assumed)	r	4%
Volatility of S	σ	30%

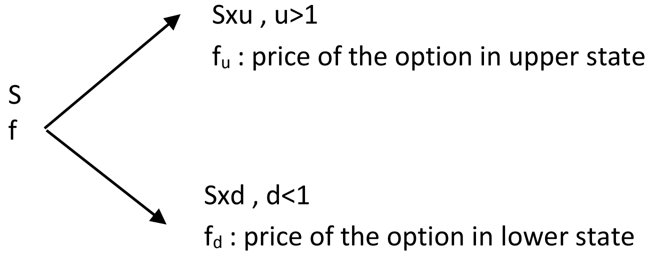
Applying Black-Scholes formula, we have

d_+	2,09568652
d_-	1,42486612
$N(d_+)$	0,981945
$N(d_-)$	0,92290205

Giving a value of the option of 254. The total value of the project is therefore 800+254=1054, whereas it requires an investment of 1000. The investment should therefore be undertaken, while the sole DCF value was advocating to stay out of the project.

The Cox-Rubinstein method for valuing options relies on a discreet case valuation, whereas the Black-Scholes method relies on a continuous time valuation.

To value the option, we chose several discreet steps at which we make an hypothesis on the price of the underlying. Between each step, the price S can go up by a certain amount u or down by a certain amount d, which enables to capture the volatility of the underlying.



Let's take a portfolio with π shares and short one option. The possible values of this portfolio at the end of the step is $S x u x \pi - f_u$ or $S x d x \pi - f_d$. This portfolio is therefore riskless when $S x u x \pi - f_u = S x d x \pi - f_d$, meaning

$$\pi = \frac{f_u - f_d}{S \times u - S \times d}$$

The value of this portfolio today is therefore $(S \times u \times \pi - f_u)e^{-rt}$ or $(S \times d \times \pi - f_d)e^{-rt}$. The price of the option price today is therefore:

$$f = S \times \pi - (S \times u \times \pi - f_u)e^{-rt}$$

$$\Leftrightarrow f = S \times \frac{f_u - f_d}{S \times u - S \times d} - (S \times u \times \frac{f_u - f_d}{S \times u - S \times d} - f_u)e^{-rt}$$

$$\Leftrightarrow f = \frac{f_u - f_d}{u - d} - (u \times \frac{f_u - f_d}{u - d} - f_u)e^{-rt}$$

$$\Leftrightarrow f = (\frac{f_u - f_d}{u - d}e^{rt} - u \times \frac{f_u - f_d}{u - d} - f_u)e^{-rt}$$

$$\Leftrightarrow \boxed{f = \left(\frac{e^{rt} - d}{u - d} f_u - \left(1 - \frac{e^{rt} - d}{u - d}\right) f_d\right) e^{-rt}}$$

We can repeat this principle for each step. To determine the value of the option, we then work backward to the value of the option today. The more steps we take for a given lifetime of the option, the closer we will get to the Black-Scholes approach.

Let's come back to our drug start-up example with five steps, each lasting for one year. Keeping the same assumptions, we have the following hypothesis for the net present value of the cash flows[25]:

1	2	3	4	5
800	988	1219	1505	**1858**
	648	800	988	1219
		525	648	800
			425	525
				344

[25] A 30% volatility corresponds to u=123% and d=1/u=81% for each step.

Now, we have to assess the price of the option at maturity. We start from year 5 (see numbers in red frame and bold letters). At this period, investing 150 would enable to increase cash flows by 50%; the option is worth P=max(1858x50%-150;0)=779. We reiterate the process for all the column. Then we apply the method exposed above to get the value of the option for year 4, 3, 2 down to now. We have the following results:

1	2	3	4	5
272	361	471	608	779
	191	262	350	460
		124	180	250
			69	112
				22

Example:

$$\left(\frac{e^{4\%\times 1}-0{,}81}{1{,}23-0{,}81}779-\left(1-\frac{e^{4\%\times 1}-0{,}81}{1{,}23-0{,}81}\right)460\right)e^{-4\%\times 1}\approx 608$$

We therefore get a price of 272. The value of the project is 800+272=1072 when it requires an investment of 1000 so it should be made.

We have seen that in both cases, the real option approach has therefore enabled to capture extra value, linked with flexibility of the project, a key element in start-ups, that wasn't comprised in the DCF approach. While the DCF focuses on only one likely scenario, the real option method takes many more possibilities into account and is therefore more adapted to encapsulate the flexibility inherent to start-ups. In practice, the method is often used to value biotech companies searching new products.

The modeling is extended by Michel Levasseur[26], who elaborates on previous real option models developed by Brennan, M. and Trigeorgis (1999), Dixit & Pindyck (1996). The idea is that a start-up creation is a two-step process. First, the entrepreneurs put money in the company to finance its first cash needs. At the end of this period, they have the choice to liquidate the company, to pursue development or to expand. At the end of the second period, we value all the possible outcomes with the real option method.

[26] "Evaluation et Coût du Capital d'une start-up", Michel Levasseur. See http://rilk.me/wp-content/uploads/2012/05/ml25.pdf

The main conclusions of his model is that the value of a start-up is directly a function of the future possible investment (I), the cash-burn rate during the development period (c), the potential value creation by monetary unit invested [ ]TEx, the diminution of returns implied by potential competition (α), of the uncertainty during the two development phases (σ1 and σ2), on the duration of the contract (T), on risk aversion (λ), and the risk-free rate (r).

He also shows with a numerical example that this valuation framework enables to find many intuitive results about start-up valuation: their high cost of capital is not linked with the low liquidity of the shares but is directly derived from the uncertainty of the cash flows, and that the valuation is much higher after the initial development phase, when risk has diminished.

d) limitations

The first problem with the real option approach is that, because of their analogy with financial options, they are priced with the same formulas, whereas the hypothesis necessary to get such formulas are far from being met in a start-up.

For example, the demonstration of the Black-Scholes formula assumes that the underlying asset can be traded (bought or sold) at any time and without cost, which allows arbitrage opportunity if there is a mispricing. It is thanks to this no-arbitrage condition that we can get to the Black-Scholes formula.

However, most start-ups are not easily tradable, unless they are listed. As we will see later on, their shareholders agreements often comprise many clauses regulating the possibility to buy or sell, or even the price at which those shares can be sold. Trading of start-ups shares is a very discreet process (trading happens only when there is a capital increase or a sale of shares) which takes very long. We are extremely far from the assumption of perfect information which is the basis of asset pricing theory.

"Although the CAPM and the Option pricing model are consistent, certain aspect of the option pricing model makes its application of valuation of new ventures difficult. One problem is that the option pricing model is derived under an assumption of market completeness and continuous trading. (…) Clearly, they do not hold for most of new ventures"[27].

[27] *Entrepreneurial finance, strategy, valuation and deal structure* by Janet Kiholm Smith, Richerd L. Smith and Richard T. Bliss

In a certain sense, the Cox-Ross-Rubinstein approach is more realistic because it is based on discrete scenarios. The method takes a strong assumption by saying that the price of the underlying can either go up and down by the same amount u or d, but to my mind, we can establish a link between the stepping-stone method and the value of the underlying asset in each scenario, depending if the stepping stone has been reached or not. The Cox-Ross-Rubinstein approach can be a good start to value the flexibility in a start-up.

A very interesting matter is however raised by Pascal Quiry[28]. “We could think the real option method is very well adapted to start-up valuation because its step-by-step way of working is very similar to the successive development steps the young start-up has to go through during its development. In practice however, the method is almost never used in this field. Writing a consistent business plan while being optimistic is complicated, but asking an entrepreneur to write downgraded versions of his business plan including a version leading to bankruptcy is counterproductive: do we want to demoralize and make him behave irresponsibly when he needs all his stamina to take up the challenge? Of course not!”.

He explains that though the real option method is theoretically correct, it fails for two reasons. The first one is that forecasting various scenarios is dispiriting for the entrepreneur when it comes to imagine cases of failures. The entrepreneur has to be full of dreams and punch to deal with the difficulty of creating his new venture. It would be useless to demoralize him beforehand by showing him how likely he is to fail.

The other one is the complexity of the approach. As I have experienced with the capital increase of my own company, investors are not at all familiar with valuation techniques. I do not have precise statistics about that, but I have experienced that many business angels have no clear notion of what a discount rate and a DCF approach are. Let alone the Black-Scholes formula.

e) Adjustments

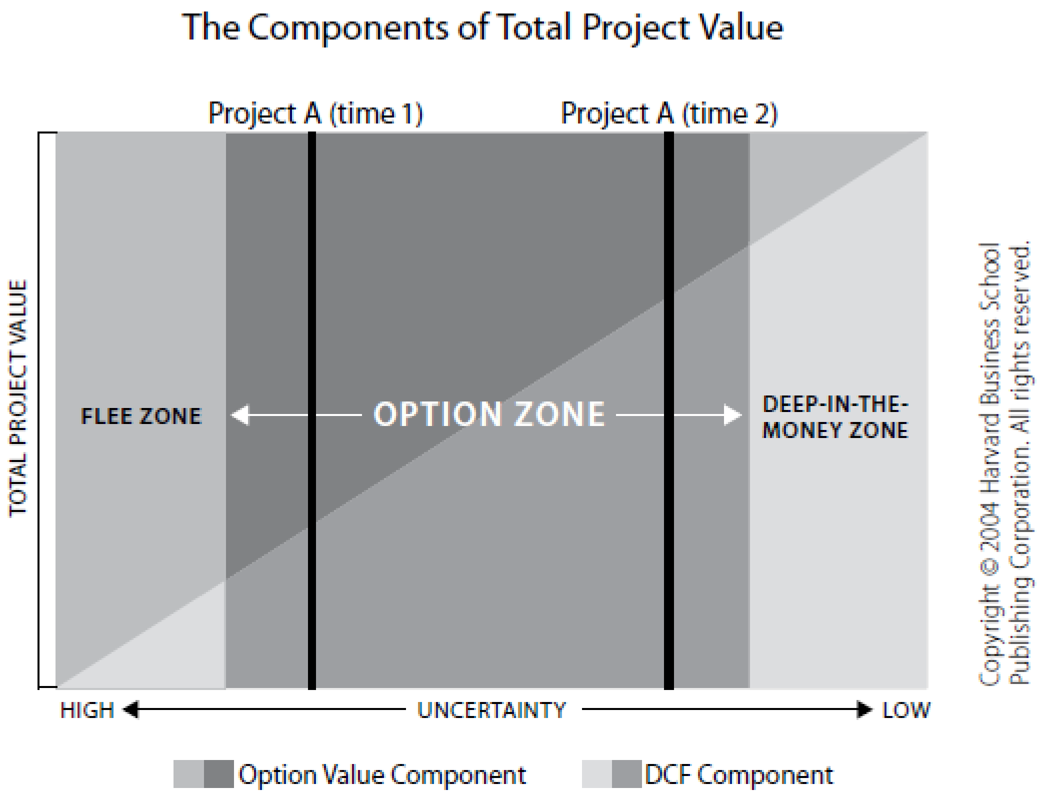
Alexander B. van Putten and Ian C. MacMillan advocate that the real option method should be applied to any project as a complement of the DCF approach:

“It seems clear to us that discounted cash flow analysis and real options are complementary and that a project’s total value is the sum of their values. The DCF valuation

[28] Translation from *Vernimen, Finance d’entreprise,* p914

captures a base estimate of value; the option valuation adds in the impact of the positive potential uncertainty”[29].

In a start-up, the DCF value will be low because of the high discount rate due to the high risk and probability of failure. But the option value will be high because the project is extremely flexible. As time goes on, the uncertainty decreases, so the DCF value increases, but the flexibility too so the option value goes down. They however insist that projects with large negative NPV for which all the value come from the option should be rejected because it might be easy to find more interesting ones. Only the projects in the dark grey zone on the graph below should be accepted.



To apply this to start-ups, we could say that the value given by the real option method is only valid as long as this result does not come entirely from the options. The DCF should account for at least a reasonable part of the total value.

[29] “Making Real Options Really Work“ by Alexander B. van Putten and Ian C. MacMillan, Harvard Business Review 2004

Two problems remain: finding volatility and adjusting for costs. Indeed, in options, the greater the volatility the larger the price of the option, but if costs volatility is high, it should destroy value. We have already seen how to estimate the volatility of cash flows, but according to them, this is not sufficient.

The reason for that is that costs are known almost with certainty while revenues are only hypothetical (this is all the more true for start-ups). Therefore, one needs to take into account an adjusted volatility that measures the gap between the uncertainty of costs and revenues:

Adjusted volatility = project volatility×(revenue volatility ÷ cost volatility).

This way, if costs are very certain and cash flows very uncertain, the adjusted volatility will be low, yielding a lower value of the option.

Most important of all they insist that the question of valuation should never be disconnected from the actual performance of the project. The imbedded flexibility measured by the real option method is only useful if this flexibility is applied in reality. This is all the more true for start-ups: "Option valuations only make sense when applied to projects that can be terminated early at low cost if things don't go well. And no valuation method will save a company that does not actually pull out quickly, if the project fails to deliver on its initial promise, and redeploy talent and funding elsewhere".

2) Broadly used in practice, the Venture Capital Method is pragmatic but has its flaws

a) Principles of the method

The venture capital method is very pragmatic, and doesn't really have theoretical justification; its broad use in practice however obliges to see how this method works and what its flaws are. The main goal of a venture capital fund being to get a return on its investment over a limited time period (generally three to seven years), the first question the venture capitalist asks himself is: "For how much will I be able to sell my stake in the company at the next round of financing?".

Such a position is very logic in the sense that as investing in a start-up is not a liquid investment, the only way to capitalize on the gains is to sell back the stake in the company at a higher valuation than the one you entered at. This can easily be done when the company has grown in the meantime and delivered some performance, but will be very hard to do if the business proves to be harder than expected.

To do so, the venture capitalist "triangulates"[30] the value of the company using the following method:

"1. They identify the maximum valuation they should consider based on their view of future valuation of the company

2. They identify the floor valuation based on the competition from other investors

3. They keep to the bottom end of this range, if possible, depending on the expectations and readiness of the entrepreneur"

To get an idea of the potential value at exit, venture capitalists try to assess the possible value of the company at the time they plan to exit, and the potential value at the next round of financing (in which they will be diluted due to the entry of a third party investor). Publicly traded comparable companies or other companies they have in their portfolio can give them a rough idea of the value at exit, as in the comparable method. The difference however is that those comparables are not so much used to get the value of the company than to grasp the potential of the market, the long-run growth rates and margins in the sector.

What is more tangible is the value that the company can attain at the next step of financing, given the milestones reached by the management. Berkery gives the following example: "While it might be extremely difficult to value the possible potential value [of a] software company that devised a new way of transferring messages from cell phones to fixed line phones, it would be a lot easier to determine the value of the company if the product were finished and the company had made one or two reference sales".

Reaching a milestone gives more visibility to the product or service sold by the company, and also gives a better view of the management's ability to achieve those goals.

When the venture capitalist has achieved this valuation at exit and next round, he needs to discount those sums to get an idea of the value he wants for the company today. In

[30] See Dermot Berkery, Raising Capital for the serious entrepreneur, p 150-160

general, given the risk taken, they look in early stage venture for a multiplication of their investment by 10 or 20 times over 6 years, in order to achieve an actual return of 6-7 times. Of course, if the project is more advanced, investors bear less risk and therefore ask for returns significantly lower. Such returns would correspond to discount rates of 40-60%, much higher than those traditionally used in DCF methods!

This gives the venture capitalist the maximum value that he would be ready to pay. But in order to be able to invest in the company, he has to make an offer good enough so that it will be accepted rather offers from competitors. It is the task of the management during the negotiation process to show that there is a lot of competition on the deal. Depending on how he feels competition on the deal, the venture capitalist will set the floor value that he will offer.

The last point of adjustment is the "readiness of the entrepreneur", namely the quality of the management. If it already has a track record and that the team is complete, it will be a lot easier for the management to push the valuation up because they have shown their ability to deliver results. Conversely, if the management has no experience or that the team lacks some key elements (e.g. a marketing or a technical director depending on the sector), then the fund will have a lot of efforts to make in order to help the company close those gaps, and will therefore require a much lower valuation.

b) Limits of the approach

We can see that this method is highly judgmental, and not really based on any theoretical justification. As Berkery puts it, "To some extent, there are analogies between real estate and early-stage company valuation. The only way in which real estate agents are able to come up with valuation for a house is through tacit knowledge regarding the neighborhood and the special features of the particular house. They use no meaningful analytical tools, yet they can often give a highly accurate view of the price at which a house will sell"[31].

However, where Berkery praises the pragmatic efficiency of this method, I do believe it can lead to valuations completely disconnected from reality.

Indeed, if we look at the internet bubble in 2000, we can see that all the factors named by Berkery to push prices up were united: very high valuation of listed comparables, very

[31] Dermot Berkery, Raising Venture Capital for the serious entrepreneur p 141.

high expectations about exit values, lot of competition to invest in the tech business… When the crisis burst up, it became obvious that many of those companies had been overvalued: Cisco Systems, which had bought 80 start-ups was obliged to pass a $2.2 billion depreciation in 2001[32].

Another flaw of this method that I have experienced myself as an entrepreneur trying to raise venture capital is that it can be hard to communicate in case of disagreement on the price. Indeed, to propose a value for my company, I had chosen a comparable named Evaneos[33] which had been recently valued a 33.5 times its earnings to derive the proposed value of my own company. The answer of the fund facing us was that the price of the comparable was too high and that investors should never have invested at such multiples in the first place…

From a theoretical point of view, this approach also has serious limits, which are pointed out by Damodaran in the *Dark side of Valuation*. His first argument is to demonstrate that there is no common measure between the venture capitalists target rate of returns and the ones they actually ask for[34].

Indeed, as we saw before, venture capitalists apply very high discount rates to discount the projects they are presented:

Stage of development	Required rate of return
Start-up	50 to 70%
First stage	40 to 60%
Second stage	35 to 50%
Bridge/IPO	25 to 35%

However, the actual returns earned by venture capitalists are far from being so high. Those are Damodaran's estimates for the returns of venture capitalists in 2007 in comparison with the NASDAQ:

[32] Cisco Systems annual report 2001
[33] Both my company, l'Hirondelle and Evaneos are in the tourism sector, operate in France, Evaneos was only four years older than my own company, and had the same margin profile
[34] Those figures are concurred by a study by Jeffrey Timmons and Stephen Spinelli in 2007

	Three-year	Five-year	Ten-year	Twenty-year
Early stage	4.90%	5.00%	32.90%	21.40%
Balanced	10.80%	11.90%	14.40%	14.70%
Later stage	12.40%	11.10%	8.50%	14.50%
All VC	**8.50%**	**8.80%**	**16.60%**	**16.90%**
NASDAQ	3.60%	7.00%	1.90%	9.20%

It is clear from this analysis that the rates of returns which are required from investments are not delivered in reality, in part because many companies don't make it to the exit phase.

Damodaran also points out three other limits of this approach:

- Focusing on future revenues and earning encourages "game playing"; the management will try to push the numbers up without really having to make the investments necessary to obtain such values, while the venture capitalists will try to push them down. "The projected value becomes a bargaining point between the two sides, rather than the subject of serious estimation".
- The multiples of comparable companies are used to estimate what trading multiples will be at the moment of exit, while in reality those comparables also have uncertain cash flows that might be very different at the time of exit.
- The high rate which are used can only have a meaning for equity investors; therefore if we use revenues or EBITDA multiples, we have to compute the cost of capital of the firm and not just its cost of equity.

In fact, such high discount rates are often only used as a matter of negotiation, as Mr Smith and Bliss explain[35]. Entrepreneurs always present business plans in which the venture develops according to plan, without problems or setbacks. If an investor is interested by the project but finds the price too high, it will be much simpler for him to use a higher discount rate to compensate for the optimistic bias rather than discuss the projections with the entrepreneur: "It is sometimes easier to build the valuation on a structure of compensating errors than to work towards unbiased value projections on which both parties agree".

[35] *Entrepreneurial finance, strategy, valuation and deal structure* by Janet Kiholm Smith, Richerd L. Smith and Richard T. Bliss. P 426.

3) The first Chicago Method, a good compromise between the two previous ones?

We have seen the limits of the venture capital method, which relies on one very unlikely success scenario but discounts it at a very high rate to compensate for this small probability of achievement.

The so-called 'First Chicago Method" was first developed by Sahlman and Scherlis in their article "A method for valuing high risk long term investments: the venture capital method"[36].

The principal idea is to determine the outcome of three main scenarios and discount them at a realistic cost of capital such as the one determined by the CAPM plus a premium for illiquidity. Then those scenarios are weighted in terms of probability, and the value of the company is just the weighted average of those scenarios.

The first advantage of this method is that it requires to think about possible outcomes of the business, and try to see how this one might evolve. It gives a better view of the potentiality of the company than the DCF or the venture capital method.

It also somehow covers some of the value imbedded in the real options through the various scenarios.

However, this method remains very judgmental, principally because the weight of probabilities has a high impact on valuation and it is impossible to get an accurate value for them. This can lead to a discrepancy between the entrepreneur's valuation and the outside investor valuation that might not be easy to close.

III) Proposition of a new valuation method

I will first present a new way of analyzing the stand-alone value of a start-up, trying to overcome as much as possible the problems encountered in the methods enumerate above. In a second part, I will present the adjustment that have to be made when going from the standalone valuation of the company to the valuation of the value of the shares for the stakeholders in the context of a capital increase: the entrepreneur and the outside investors.

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[36] Harvard Business School Note 9-288-006 (2009). See http://hbr.org/product/A-Method-For-Valuing-High/an/288006-PDF-ENG

1) Proposition of a new valuation method

a) Standing point and objectives of the method

Before presenting the method itself, it is important to summarize what we have learnt so far and then set the priorities of the method I have elaborated. I will afterwards present the method on an example.

First, classic valuation method have the merit to be simple, but they can hardly give a consistent value of a start-up because they either focus on one unlikely scenario or compare the company with others that in fact are not so comparable.

On the contrary, the real option method is in theory perfectly relevant to take into account the flexibility of start-ups, but its complexity and strong assumptions make it seldom used in practice.

Finally, the practitioners' methods (i.e. the venture capital and the first Chicago method) are a mix of both, but their lack of theoretical justification and the rough assumptions taken can sometimes take the price far away from the firm's value. Besides, they are usually biased towards the interest of the VC fund at the expense of the entrepreneur.

Based on those observations, we need a method that will partly rely on traditional valuation techniques in order to be understood by potential investors but those results will have to be adjusted to take into account possible shifts in the firm's strategy and the high level of risk present in a start-up.

b) Presentation of the method

In the real options method presented above, the possibility of shifts in the business model was taken through the presence and valuation of imbedded real options, such as growth options.

But we can also consider the whole company as an option, an approach that has been increasingly developed in the real option framework. Indeed, for an indebted company, if a firm is not able to pay for its debt at maturity, shareholders will decide to leave the company to debtholders by not repaying the debt and cutting their losses. In a certain sense, it is as if debtholders had sold a put option to shareholders that could be exercised if the enterprise value at maturity (debt and equity) is inferior to the debt to be repaid.

Conversely, the equity value can be seen as a call on the company with a strike price equal to the debt to be repaid: if the enterprise value at maturity is superior to the debt, shareholders will pay it back and get access to all the enterprise value. In this framework, we can then value the whole company by the Black-Scholes formula:

$$E = V\Phi(d_1) - De^{-r\tau}\Phi(d_2)$$

Where

Element	**Notation**
Enterprise value	V
Equity value	E
Net debt (book value)	D
Volatility	σ
Risk-free rate (discr.)	r
Time to maturity	τ

And

$$d_1 = \frac{\ln\left(\frac{V}{E}\right)+\left(r+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = \frac{\ln\left(\frac{V}{E}\right)-\left(r+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}$$

Φ the cumulative distribution function of a normal centered variable.

In clearer terms, $\Phi(d_2)$ is the probability that the firm doesn't go bankrupt.

As we have said, the most crucial parameter to estimate here is the volatility of cash flows, or the volatility of the enterprise value σ. To do so, the idea is to look at comparable listed firms in order to assess the volatility of their equity σ_E. The Hull-White formula then shows that we have:

$$\begin{cases} E = V\Phi\left(\dfrac{\ln\left(\frac{V}{E}\right) + \left(r + \frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}\right) - De^{-r\tau}\Phi\left(\dfrac{\ln\left(\frac{V}{E}\right) + \left(r - \frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}\right) \\ E = \Phi(d_1)\dfrac{\sigma_V}{\sigma_E}V \end{cases}$$

Which we can solve for V and σ_V. This will enable to give the volatility of cash flows in the sector.

The idea is then to compute the enterprise value with a DCF approach to get a first approximation of the enterprise value and therefore of the equity value.

Considering the volatility of the enterprise value as a proxy for the riskiness of the sector, we can use Amada's formula transposed to real option in order to deduct the volatility of the equity value of the firm to value:

$$\sigma_E = \frac{\sigma_V}{1 + \frac{B}{E}}$$

Finally, we can deduct the value of equity using the Black-Scholes formula with the volatility we have found.

c) example

We are going to value Theraclion, which is a biotech company that was listed on the 24th april 2014. We will use the actual market cap as a way to check the consistency of the methodology employed. Qualifying Theraclion as a start-up is not an euphemism, as long as it has only generated 14500€ of turnover so far, for 8.8m€ of accumulated losses.

To value Theraclion, I have looked at other companies in the biotech industry that have been listed recently and have retained the followings: Nanobiotix, Implanet, Spineguard and Visiomed.

All those companies are quite young, and have always had negative operational cash flows. They therefore correspond to the definition of start-ups that we have given before. We can therefore consider that their risk is comparable. They have been IPOed between one and two years, which enables to have enough market data to analyze them. From this market data and companies annual reports, I extracted the following information:

Element	Notation	Visiomed	Implanet	Spineguard	Nanobiotix
Market cap (M€)	E	9,86	38,61	41,19	189,66
Net debt		-0,43	1,01	-4,14	-10,93
Enterprise value	V	9,43	39,62	37,05	178,73
Face value of debt	D	0,79	5,91	2,25	1,43
Volatility of equity	σ	72%	14%	48%	99%
Risk-free rate (discr.)	R	2,04%	2,04%	2,04%	2,04%
Time to maturity	τ	1	2,5	3	2,7

From which we get the following Black-Scholes elements:

Notation	Visiomed	Implanet	Spineguard	Nanobiotix
r continuous	2,02%	2,02%	2,02%	2,02%
d1	4,5208	37,0910	9,6576	6,3010
d2	3,8018	36,8670	8,8273	4,6851
Φ(d1)	0,99999692	1	1	1
Φ(d2)	0,99992817	1	1	0,9999986
Equity value by Black-Scholes	8,65	34,00	34,93	177,38

Using the Hull-White approach, we also get:

Element	Notation	Visiomed	Implanet	Spineguard	Nanobiotix	Average
Volatility of EV	σV	69%	12%	47%	99%	57%
Enterprise value	V	13	44	44	192	

We see that Implanet’s volatility is surprisingly low. I have therefore retained the average of the three other volatilities as a measure of the volatility that will be applied to Theraclion: 72%.

Based on a DCF approach, with the following assumptions, we get the results below:

	31/12/2013	31/12/2014	31/12/2015	31/12/2016	31/12/2017	31/12/2018	31/12/2019	31/12/2020	31/12/2021	31/12/2022	31/12/2023	31/12/2024
EBIT(1-IS)	-3 046 K€	-1 893 K€	-741 K€	412 K€	1 565 K€	2 718 K€	4 294 K€	6 312 K€	8 584 K€	10 730 K€	12 232 K€	12 599 K€
Investissements	-231 K€	-238 K€	-244 K€	-250 K€	-256 K€	-263 K€	-269 K€	-275 K€	-281 K€	-288 K€	-294 K€	-300 K€
+D&A	77 K€	97 K€	118 K€	138 K€	158 K€	178 K€	199 K€	219 K€	239 K€	259 K€	280 K€	300 K€
-Δ WCR	710 K€	10 K€	40 K€	150 K€	400 K€	414 K€	588 K€	753 K€	848 K€	801 K€	561 K€	137 K€
FCF	-2 490 K€	-2 024 K€	-827 K€	450 K€	1 867 K€	3 047 K€	4 812 K€	7 009 K€	9 390 K€	11 503 K€	12 779 K€	12 736 K€
Discount rate	1,00	0,89	0,80	0,71	0,63	0,56	0,50	0,45	0,40	0,36	0,32	0,28
DCFF	-2 490 K€	-1 805 K€	-658 K€	319 K€	1 182 K€	1 721 K€	2 424 K€	3 149 K€	3 763 K€	4 112 K€	4 075 K€	36 313 K€
Enterprise value	52 105 K€											

Element	Value
Perpetual rate	3%
Tax	33%
Risk-free rate	2,04%
Market premium	5,50%
Beta	2,04
Cost of equity	13,3%
Cost of debt after tax	1,50%
Cost of capital	12,11%
Enterprise value	**52,1**
Equity	**46,9**
Net debt	**5,2**

Hence, using the Amada formula, we know that Theraclion's volatility of equity will be around 65%.

The Black-Scholes formula applied to Theraclion then yields 48,7M€ for the equity value.

Element	Value
Net debt	5,2
Enterprise value	53,7
Face value of debt	5,58
Volatility of equity	51%
Risk-free rate (discr.)	2,04%
Time to maturity	2,7
r continuous	2,02%
d1	3,27219221
d2	2,43289467
Φ(d1)	0,99946642
Φ(d2)	0,99251067
Market value of debt	5,27
Equity by Black-Scholes	47,2

As a sanity check, we see that we are not far from the actual market cap which was 51,69 M€ on the 01/05/2014.

d) Comments

In this example, we get close results between the DCF value and the value of the equity as restated by the real options for several reasons:

- The maturity of Theraclion's debt is quite short. Because it was not possible to determine exactly the maturity of its long term debt, it was approximated by the average of the other comparable companies' debt maturities. The longer the maturity, the larger the difference.

- Theraclion has a low level of debt, as all the companies in its sector, and as many start-ups. The larger the debt, the larger the difference because of a discount in the market value of debt due to the risk of bankruptcy.

- The discount rate obtained by the DCF is surprisingly low for a start-up, meaning that the comparables companies that were used to compute the discount rate were more mature than Theraclion. The volatility of equity obtained by my calculation is however much larger than that of a traditional company and therefore better states the level of risk present in a start-up.

The method presented above relies on traditional valuation techniques but adjusts them to the specific level of risk in start-ups. In that sense, it is a more realistic approach for start-ups valuation. Besides, we see that it gives a higher value for the equity because it takes into account volatility.

2) Special features of a start-up shareholder's agreement have to be taken into account into the post-money valuation

a) The heterogeneity of the cost of capital

However, the standalone valuation is not what matters most. Indeed, the moment when the question of a start-up value comes into play is during the various rounds of financing that take place. As we have seen, the specificities of start-ups make those rounds of financing take the form of capital increases. We therefore have three kinds of players at stake: the entrepreneur, existing shareholders and potential new shareholders.

Smith develops the idea that the cost of capital can't be the same for the entrepreneur and outside investors[37]. Indeed, the CAPM formula which determines the cost of capital in the DCF model is determined on the assumption that the investor is fully diversified and can therefore build a portfolio offering an optimal hedge against market risk.

This is clearly not the case for the entrepreneur, who commits a significant part of his wealth and of his time to his venture. "Entrepreneurs' portfolios are less well diversified than those of non-entrepreneurs. They hold less wealth in liquid assets, bonds, public equity and housing; they hold more wealth in business assets and nonresidential real estate" (Gentry and Hubbard [2004]).

For the entrepreneur, the cost of capital is therefore twofold: the first one is financial and the second one is human. Indeed, when engaging into a venture, the entrepreneur generally has to cease other activities such as his studies or his job. The first point of comparison is therefore the expectation of what he could earn with the venture and what he could earn by taking no risk in remaining in his current position. The second point is that in case of failure or success, because he has a significant amount of his wealth in his business, the entrepreneur is highly exposed to the specific risk of his business that is by nature non-diversifiable.

Smith takes the following example of an entrepreneur having a total wealth (financial and human capital) of $300 000 and a project requiring $100 000 with the following prospects in one year:

Scenario	Probability	Year 1 payoff	Return
Success	1/3	200 000	100%
Likely	1/3	125 000	25%
Failure	1/3	53 000	-47%

The expected return is therefore 26% and the volatility of the project 60%.

If we assume that the risk-free rate is 4% and the return on the market portfolio is 12%, a correlation of the project with the market of 0,5, and a standard deviation of market

[37] See chapter 11 of *Entrepreneurial finance*

return of 15% the required rate of return for a diversified investor is given by the CAPM: 4%+(0,5x60%/15%)=20%.

But the entrepreneur has 1/3 of her wealth in the project, and can only have 2/3 in the market, the volatility of his portfolio is:

$$\sigma_{port} = \sqrt{\left(\frac{2}{3}\right)^2 \times 15\%^2 + \left(\frac{1}{3}\right)^2 \times 60\%^2 + 2 \times \left(\frac{2}{3}\right) \times \left(\frac{1}{3}\right) \times 15\% \times 60\% \times 0{,}5} = 26.5\%$$

The return on his risky portfolio should therefore be according to the CAPM:

$$r_{port} = r_f + \frac{\sigma_P}{\sigma_M}\left(r_M - r_f\right) = 4\% + \frac{26{,}5\%}{15\%} \times (12\% - 8\%) = 18.1\%$$

And finally, we deduct the required rate of return on his project:

$$r_{port} = x_p r_P + x_M r_M \Leftrightarrow 18.1\% = \frac{1}{3} r_p + \frac{2}{3} 12\% \Leftrightarrow \boxed{r_p = 30.3\%}$$

The required rate of return for her project is therefore 30.3%, much higher than the rate of return for the diversified investor and the rate of return of the project. The project should therefore not be undertaken for the entrepreneur while it would be profitable for an outside investor! Of course, we can show that the less diversified is the entrepreneur, the higher would be the required rate of return.

This diversification depends on how much the entrepreneur actually invests financially in his company, but also on the time and effort he commits to it.

This is problematic, because it means that for a given stream of free cash flows, their discounted value would be different for the entrepreneur and the outside investor. On this basis, how to agree on a potential value?

But it also means that by bringing other investors into the venture, the entrepreneur will lower his required rate of return (because a smaller fraction of his wealth is invested in the project), and hence he can increase the NPV of the project for him. The right allocation of returns for all parties is at the heart of the deal structuring in start-ups' shareholders agreements, and we will see how those elements have to be taken into account to correct the value of the company for each party.

b) How to value contractual clauses in the shareholder agreement?

It is necessary to distinguish two cases: the entrance of new investors with a goodwill and the entrance of a new investor without goodwill.

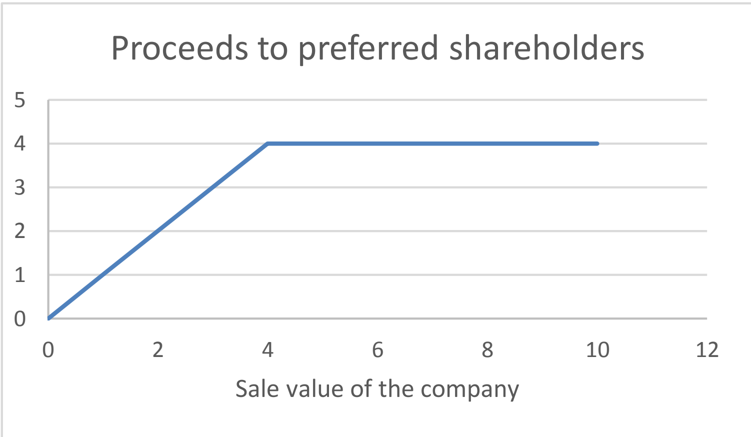
In order to reward and give an incentive to the entrepreneur for his full commitments, outside investors very often enter in the capital of new ventures at a much higher price than the one the entrepreneur entered at. There is often an important goodwill, which creates an asymmetry of interests.

Indeed, if the outside investor enters at a very high price and that the entrepreneur quits the company selling his shares, the entrepreneur might make a profit while the investor would lose money. The same thing can happen if the entrepreneur lets new investors enter at a lower price than previous round investors: first round investors would get diluted and it would be hard for them to get back the value of this investment.

The main clauses linked to the presence of a goodwill during the initial investment are preferred stocks and ratchet clauses. Preferred stocks define a distribution of revenues between the investor and the entrepreneur.

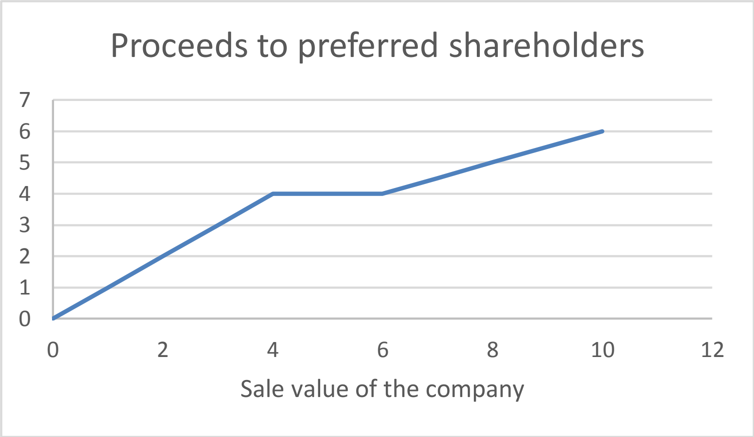
Some examples of this are[38]:

- **Redeemable preferred stock**: up to a certain value, new round investors would get a preemptive right on all the proceeds of the sale of shares, but nothing after. We recognize the pay-off profile of a short call option on the firm's equity value.



[38] Dermot Berkery, *Raising Venture Capital for the serious Entrpreneur*.

- **Convertible preferred stock**: Up to a certain level, all proceeds go to preferred shareholders, and then, after another level, gains are shared equally between both the entrepreneur and preferred shareholders. We recognize the combination of two calls: one short with a strike price of $4M and one half of a long call with a strike of $6M. Ratchet clauses also have those kinds of payoffs profiles but with the strikes being equal.



Berkery doesn't mention it, but we can see that such stocks are combinations of options, and they can therefore be priced as such. More generally, it should be possible to assimilate any kind of revenues repartition between preferred shareholders and common shareholders by options payoffs. As for financial options, we can identify all their characteristics except volatility, which have to be agreed upon by both parties. However, there is no reason why we couldn't apply the same method for calculating volatility as in the real option method.

The second possibility is the capital increase without goodwill. In this case, everyone pays the same price for the shares, so the entrepreneur gets very diluted. As a compensation, he is granted stock-options that he can exercise to be reluted if the share price reaches a certain level. This has the advantage of enabling the outside investors not to suffer from an overvalued stock price: if the company faces difficulty, current investors will be more willing to put some more money on the table and all interests will be aligned. However, being extremely diluted provokes a great risk of demotivation on behalf of the entrepreneur, who would not really feel that the company belongs to him anymore, thus endangering the company.

I would like to further push an idea I found in Mr Quiry's Vernimen. He explains that if the entrepreneur believes in his valuation, it is impossible for him to refuse a ratchet clause that would enable the investor to recover at least the price he paid to enter the company.

But I think that conversely, we can say that if the outside investor believes his valuation to be fair, he should have no argument to refuse a capital increase without goodwill and granting the management call options with a strike price equal to this valuation. Indeed, if he sets a strike price for the options so high that he thinks it will never be reached, then the entrepreneur is completely demotivated, so in the end the shares are worth nothing for both of them. But if he accepts a low strike price he will be sure to be diluted so this is not a good strategy either.

Therefore with this reasoning, the price of the company should be comprised between the price at which the entrepreneur is ready to accept a ratchet clause (maximum price) and the price at which the outside investor would be ready to propose call options to the entrepreneur in the framework of a capital increase without goodwill.

Conclusion

Valuing a start-up comprises many problems, the first one of them being that it is extremely hard to tell what the future of the company will be, or more prosaically if it will survive at all in the coming years. Because this level of risk has nothing in common with that of mature companies, it is extremely hard to correctly use the DCF or the multiples approach in order to appreciate their value.

But it is not a reason to rely on rule of thumbs or rough techniques, such as the Venture Capital Method or the First Chicago Method to value a company.

Indeed, when we looked at what made start-ups special, we saw that what made start-ups really special compared with other companies was that they are able to evolve in many ways. Those wide arrays of possibilities can be well captured by the real option method, when considering those redeployment possibilities as imbedded growth options, or abandonment options… The high level of risk can be easily captured through the notion of volatility, which can be easily computed on comparable listed firms.

It is however even more enriching to consider the whole company, and more precisely its equity as an option. Indeed, this enables to capture the fact that in case of default, which is more likely in start-ups than in other companies, the shareholders will abandon the company to its creditors. The claim that shareholders have on equity is nothing else than a call on the enterprise value, and which can therefore be computed using the Black-Scholes formula.

In this framework, we have seen that it was possible to get a first approximation of the company's value with traditional techniques (preferably the DCF, which has more theoretical justifications that the multiples) and then to adjust the price thanks to the real option method. We have also seen that in a start-up, the claims on equity are diversified, but that they can also be seen in terms of options. This enables to adjust the price for each shareholder depending on his claims. Real options therefore provide valuable insights about start-ups valuation.

But whatever the outcome of the valuation, one should not forget that the final price will be set by the quality of the project, the confidence that investors have in the management to deliver their business plan, and by the offer and demand.

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