FINANCIAL BUBBLES AND RATIONALITY

Mémoire – Majeure Finance

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Content

Abstract

In spite of the denunciations of public opinion against market agents' irrational behaviours in the event of financial crashes, we show in this paper that financial bubbles may be rational. Basing on the existing research, we set up a simple model based on the psychology of market agents showing that financial bubbles may be rationally generated.

By showing that the behaviour and the valuation of a firm are dependant on the beliefs of a few "convinced" investors in the market, our model accounts for the rational generation of bubbles and the high volatility of prices that comes along.

Special thanks to Mr Lovo for his support and his help in introducing me to the financial research dealing with rational herd behaviour and financial bubbles and in setting up with me the financial model to follow.

Introduction

Without going back too far away in time, it seems that financial booms and crashes are not a new phenomenon. The financial crash of 1929 is noteworthy in this respect. However along with the development and globalisation of financial markets, booms and crashes seem to have broken out more and more frequently for the past decades and to have spread quicker and quicker to all financial markets worldwide. They furthermore seem to have gained in intensity. Is it not today mutually agreed that the current financial crisis is the strongest ever been experienced, strongest than the 2000 crisis and even strongest than the 1987 crisis – of course this statement is lacking the necessary detachment required to judge objectively but the sharp intensity of the crisis is still not to be denied.

Firms' fundamentals alone cannot account for the brutal rises or drops in prices one may observe during periods of booms or crashes. This is undoubtedly one reason why so many journalists, politicians, even investors have then started decrying the irrationality of financial markets. Astonishing is for example to hear in the present times the criticisms of public opinion regarding the irrational behaviours of market agents, who allegedly behave in herds and whose irresponsible behaviours would have caused the current financial mess. These hasty and simplistic conclusions are however of no help in trying to understand the mechanisms of financial bubbles and consequently in trying to predict them in the future. Furthermore they cast full doubts on the way financial markets function, as these are precisely the rare kinds of markets to be supposed to operate on a rational basis.

It has hence seemed very interesting to me under the current circumstances to examine the relationship between financial bubbles and rationality. My objective here is not at all to exonerate the responsibilities of some in the context of the current crisis but rather to try to understand how financial bubbles can be generated in a rational market. A large part of research has so far taken an interest in this issue and has come to the conclusion that financial bubbles may be rationally generated under certain circumstances. The objective of this paper here is not to describe the results provided by the existing research but rather to try to deepen the issue by trying to set up a simple model of rational bubbles focusing on the psychology of investors. For this purpose we relied on the analysis *Credit Ratings as Coordination Mechanisms* by Boot, Milbourn and Schmeits (2005) that we extended to consider the influence the psychology of some investors may have on the rest of the market.

In this paper we will thereby first try to show the limits of an approach that systematically denounces the irrationality of markets as the cause of financial bubbles by considering the current crisis in particular. Then after seeing very quickly how financial research has tackled the issue of rationality and financial bubbles, we will finally try to set up a simple model to address this issue.

I. "The Irrational Exuberance of Markets"

The current financial crisis and the financial bubble have often been denounced as irrational

Witnessing the current financial crisis, some people have tried to find explanations for the crash of markets. They often call for the growing complexity of the financial system: the crisis would have originated in the globalization of the financial system, the disintermediation of operations, and the financial innovation that has used more and more complex mathematical tools. But even if experts have identified the basic phenomena that triggered the financial crisis (in very broad outline, the subprime crisis spreading to the whole financial system through asset securitization), it seems that nobody has yet been able to identify what specifically has gone wrong into the system, that is what the particular failure of the existing system is the financial institutions and the governments should more specifically act on.

In fact, since the beginning of the crisis in the summer of 2007, interest rates have been largely reduced (US interest rates decreased from mid-2007 5.25% to nearly zero (0.25%) in December 2008, Euro interest rates from 4.0% to 2.5%[1]), huge amounts of money have been injected into economies throughout the world to cope with the increasing liquidity shortage and some urgency plans have been implemented to recapitalize major banks. But despite all these actions, the crisis, which has now spread to nearly all the layers of the economy, still remains and, more worryingly, seems to get stronger: major stock exchanges keep following a decreasing trend and renowned international banks keep on fearing about their remaining independent (let us for example think of the not so singular case of Morgan Stanley, whose regular publication of bad results for the past twelve months have kept fuelling uncertainty around the stock since then). Even more worryingly, nobody really seems to know how long the crisis should last.

Hence, despite the fact that the basic mechanisms of the current crisis have been identified, the apparent inefficiency of the governments' actions and the uncertainty shared among experts about how long the crisis should last have let people start stressing out the irrationality of this crisis. As a financial analyst at Euler Hermes turns it out: "Pourquoi personne ne parvient à localiser la faille? Comment en est-on arrivé à une situation aussi paradoxale ?

[1] Source : Newsrun

Nous manquons d'explications." Not so seldom in fact has the press denounced the irrationality of this crisis over the last few months. One could in fact read titles like "La bourse bascule dans l'irrationnel" in Challenges, a French economic magazine in October 2008; Libération was talking about how irrational the system was in January 2008. Interestingly these kinds of statements have appeared in all kinds of press: in popular as well as more economic and financial–oriented press, in politically left-oriented as well as politically right-oriented press, underlying how this idea had become some kind of commonly accepted feeling.

In this paper, we will more specifically focus on the financial crisis and not address the contagion of the crisis to the whole economy. We do not mean to give an exhaustive overview of the financial crisis, but rather to gather some elements to question the commonly accepted idea that this crisis is all the more severe as it has been driven by irrational actors and is a crisis of a financial system that has become completely irrational. This idea is not really a new one and has seemed to reappear every time a severe financial crisis has taken place. Didn't Alan Greenspan, then chairman of the Federal Reserve Board in Washington, denounce in December 1996 the *irrational exuberance* of equity markets? But the strength of the current crisis has left today even more room to those who denounce such irrationality.

It is in fact often underlined how prices have recently been disconnected from fair values and their evolutions have undermined every forecasting exercise

First, it may be useful to define what is meant by "irrational". According to the Collins dictionary, something is irrational when it is inconsistent with reason or logic or in some way absurd. In this sense, it may be easier to understand why one of the first elements that are stressed to denounce the irrationality of the system and to explain the crash of markets is the decoupling which seems to have more and more strengthened for the last twenty years between the financial and the production or economic spheres. Today indeed as the worldwide production of goods and services produces $500,000 bn (2008), the sole ongoing operations on derivative products on financial markets amount to $700,000 bn that is nearly 1.5 times more.

However theoretically, as per the theory of market efficiency, such a decoupling should not exist: the share price of a firm and its intrinsic value should be equal, as the share price should

aggregate all information about the firm. Yet, evolutions of equity markets have shown that the two values can strongly deviate from each other as the share price depends from supply and demand on the market which do not only rely on the firm's fundamentals. This strong deviation is a common argument for those who denounce the irrationality of the financial system, as stock prices do not then only reflect the firm's fundamentals and the available information and hence cannot entirely be explained. This deviation can move prices up as well as down, as can be currently observed. Terms such as "crazy" and "irrational" have hence been recently heard in many cases from executive directors who could not but witness the steady decrease in their share price. In this respect, Jacques Gounon, Eurotunnel's executive director recently said "C'est totalement irrationnel, je refuse de comprendre" or Henri Proglio, Véolia's executive director "Mon action vaut au moins €40 ! ». Besides, a constant argument that reappears every time a bubble has arisen is that market operators act like lemmings and imitate each other actions instead of relying on public information and companies' fundamentals to make their decisions. They are consequently denounced as irrational and held responsible for bubbles, which grow with no connection to companies' fundamental values, and for the crashes that follows next.

This crisis may after all be considered as irrational because financial markets have not performed in line with what had been anticipated a year and a half ago. Market operators have not been able to forecast such a crisis and hence may now regard it as irrational as it has not been a logically anticipated phenomenon. It is interesting to note that denouncing these markets as irrational implicitly assumes that actors consider markets as rational initially. In fact, financial innovation, the automation of major exchanges and the fact that the financial sphere today gathers high-skilled specialists may have contributed to convey a commonly accepted feeling that financial markets are in someway "mastered" and that its evolutions or at least its trends may be forecast. The simple fact for example of creating an index such as the CAC 40, gathering the biggest French stocks implicitly assumes that these 40 first stocks are expected to remain in any time the 40 biggest on Paris exchange. Of course this has been the case most of the time since the index was created, but interestingly things do no longer prevail in current time. On 2 January 2009, 9 out of these 40 stocks did not belong to the top 40 group: Michelin and Renault dropped to the 42nd and 43rd position, Vallourec, ST Microelectronics, Cap Gemini, Lagardere and Alcatel-Lucent positioned between the 48th and 59th position, and Air France-KLM and Peugeot dropped to the 69th and 70th position. No

wonder that the fact that this crisis has reassessed the way things have been organized and understood until then may push some to denounce it as completely crazy and irrational.

These denunciations which decry market operators as acting on irrational motives are however worrying

More generally it can be noticed that things are denounced as irrational only when no more explanations can be found. The current financial crisis has indeed to some extent questioned the way people and even experts understood and organized markets. But quickly denouncing the crisis as the consequence of the irrationality of markets is not only an intellectually weak idea, but it is also, I think, an intellectually dangerous one. It first appears as intellectually weak because this explanation is generally waved by default, because no other convincing explanation can be found. And more worryingly, I think it can become dangerous, as waving this argument can prevent people from thinking more deeply and seriously about the real reasons of this crisis. As a consequence, other similar bubbles may not be forecast in the near future. Maybe the simple fact that for the past thirty years, increasingly numerous financial bubbles – followed by crashes – have occurred and have similarly been denounced, may be explained by the fact that no elements explaining financial bubbles had really been popularized, as the conviction that bubbles were irrational prevented public opinion to think someway further. I do not question the fact that every financial bubble may each have its specific causes, but denouncing them as irrational without trying to understand if some common causes or behaviours can explain them is worrying.

The common argument underlying all these denunciations of the irrationality of the current crisis – and of the other ones before - is simply that although the finance sphere is now driven by more and more complex mathematical models and although its tools tend to get more and more automated, it remains operated by humans. As a consequence, it must be recognized that irrational behaviours are very likely to happen in financial markets, as humans are naturally not only driven by rationality. They are all the more so likely to happen in the sphere of banking as the environment in which operators work is generally known to be very stressful and driven by short-run performance. Jobs are generally remunerated by generous salaries, which may have however reinforced irrational behaviours – driven for example by the fear of losing one's job. The recent Kerviel affair at Société Générale can attest of such environment - Those who denounced irrationality of operators on financial markets specially displayed this

affair. The Internet has furthermore given to these “irrational” behaviours an increasing impact, giving stronger impact to false perceptions and herd behaviours.

Furthermore, if traders may be irrational, his/her advisors are also allegedly driven by irrational behaviours and may hence reinforce the trend. Although financial information gets more easily accessible and more thorough, some research papers such as the 1995 empirical study carried out by David Dreman and Michael Berry has shown that the gap between analyst’s forecasts and companies’ financial results has steadily increased since the 1970’s. We could assume that technical complexity may make companies’ results more difficult to forecast or released information may be harder to interpret. However the simple facts that analysts are generally specialists of a particular sector and/or of particular firms and that the Internet and the reinforcement of stock-exchange regulations on companies’ information release have made public information significant, can make us uncomfortable about this hypothesis. Another way to interpret these results would be to say that analysts compute their forecasts by relying more and more on the perception and the intuition they have of the company and less on publicly released data. In this sense, they would not behave fully rationally, if by "behaving rationally" we expect them to give much more weight on publicly released information than on their personal intuitions which rather come under psychological order. The reality is unfortunately not so simple as public information may be subject to multiple interpretations.

Yet these denunciations take the perfection of markets as granted, which may appear as a simplistic assumption and question the relevancy of the irrationality arguments

However, these kinds of arguments about the irrationality of the crisis do not seem fully satisfactory because they consider a very restrictive definition of what rationality is. In fact, it seems that they consider that a rational decision should be a decision made without taking into account one's intuition or belief. They seem to consider that publicly released information is in some way perfect, i.e. fully understandable for any specialist. As a consequence, each operator would be considered as rational if his decisions are only based on public information, which should be considered as totally reliable since it has been released by the company itself. Each fully rational operator should operate independently from other operators and use the same public available information to make his/her decisions, so as to compute forecasts that

are as close as possible to reality. As a consequence, each operator should reach the same conclusion for the same problem. It should indeed be useless to use another operator's results, as these should have aggregated exactly the same amount of information as the initial operator. The idea that publicly released information is plenty enough and does not leave any room for interpretation so that it enables specialists to exactly forecast future results of a company or future stock price evolutions, is commonly accepted. It may certainly be linked in someway to the tremendously increasing amount of publicly available information – for regulatory reasons as well as firms' effort towards more and more transparency - and its higher availability - due to the development of new ways of communications like the Internet in particular. So theoretically – and this idea is commonly spread among the population - there should be no room in financial markets for operator's psychology.

But this assumption is somewhat puzzling when we consider how trust and visibility are fundamental issues in finance. The current situation can attest of such a fact, as the crisis reached a peak as soon as banks started to mistrust each other, drying up all available liquidity on the market. And today the situation is all the more worrying as nobody seems to have improved its visibility: no expert really knows yet what is to happen next and when the whole situation should calm down and stabilize. Besides, one should not forget that market operators remain specialists who generally graduate from the best schools and exercise highly remunerated jobs. It can be true that some may act in some incomprehensible ways but it seems completely insane to assume that they all act completely irrationally and still earn important amounts of money in acting so. So how can we understand that markets rely so much on psychological factors and at the same time are operated by people who must know what they are doing?

It appears actually that market operators cannot purely act basing on public information without taking their beliefs and interpretations into account just because they cannot get access to all available information. They need to make rational assumptions about certain data – for example about the composition of the market – and only then make choices and behave in the most rational way possible. It is hence necessary to reconsider the definition of rationality we previously gave. A market operator can rationally act if, considering the rational beliefs and hypotheses he needed to make about the market, he optimally behaves. Because he does not have access to all available information, he must make some coherent hypotheses in order to act. As hypotheses and though these may be rationally initiated, his

beliefs may be wrong. However his behaviour cannot be considered as irrational as public opinion does believe. Extending the definition of rationality is hence necessary to better understand the way markets may behave. This may give us a new and more interesting perspective to understand the current crisis and should allow us to keep away from all the popular arguments of irrationality which finally appear as very weak and not very helpful to understand and prevent similar crises in the future.

II. Rational bubbles have already been studied

When considering the commonly accepted efficient market hypothesis (EMH), it may be difficult to understand why and how financial bubbles, i.e. high volume trades occurring at a price that is significantly higher than should be according to fundamentals, can happen. In fact, as the EMH states that stock prices should equal fundamental values, no bubble should occur. But, as we have already underlined it above, this theory assumes that all information is publicly available to all market participants, which can be questionable in practise. That is why a large part of financial literature has striven to readdress the issue of financial bubbles and rationality and to understand if not the logic, the mechanisms of financial bubbles. This literature has particularly interested in herding behaviours to explain why in the context of financial bubbles, prices significantly moved away from their fundamental values. Very interesting is the literature that has tried to model and explain "rational herding" phenomena in relation to the formation of bubbles. As the current crisis cannot only be held as irrational, as we have tried to demonstrate above, we will specifically focus on this literature of "rational herding" so as to understand how bubbles can be rationally be generated. My objective here is not to present and describe all the literature of rational herding. I will focus on the informational approach of these theories and shortly present the fundamental results this literature has produced in order to get a snapshot of where research stands at this time. From this point, we will try to set up a model basing on an informational approach, which may account how bubbles can occur and when.

The simplest explanation of rational herding may be informational cascades. This theory was introduced by Bikhchandani, Hishleifer and Welch (1992). This model assumes that agents can only observe each other's actions on the market (buy/sell). They have no access to other

agent's private information and they trade sequentially. The idea of the model is to show that agents gain some information after observing past actions of previous agents, to the point that they can rationally ignore their private information and decide to act in accordance with the crowd. It can be shown that when the number of past buy orders (sell orders) is greater than the number of sell orders (buy orders) by more than two, it is optimal for the agent to ignore its private signal and to buy (sell), hence starting an informational cascade. Cascade can consequently start on the basis of very little information. They happen to be very fragile as any little information may shatter the cascade down and as a consequence produce very instable equilibria.

However this model makes the assumption that prices are fixed, which may appear as a problem when we try to understand the generation and the mechanisms of bubbles on financial markets. These bubbles can be observed ex post when it appears that prices have significantly moved away from their fundamental values. The assumption of fixed prices appears hence as unsatisfying in our attempt to understand how financial bubbles may rationally be generated. Avery and Zemsky (1996) address this issue and relax the demanding hypothesis of fixed prices. Considering the same framework as Bikhchandani, Hishleifer and Welch, they show that no herding can occur. In fact, as prices incorporate all available information, any investor who has private information has an "advantage" on the market and acts accordingly. For example, if his information signals that the value should be higher than the current price, he will buy. The price will then adjust accordingly to reflect all the available information. As soon as an investor perceives that the price differs from the fair value, he will act accordingly and the price will converge to the fair value. Hence herding is impossible in such a framework, i.e. when there is only uncertainty about the value of the asset in the market.

But then Avery and Zemsky introduce a new hypothesis, which is much more realistic and is much more interesting for us who try to understand how the current situation of a financial bubble that crashed could have rationally happened. Considering the previously described framework, they introduce what they call "uncertainty about the composition of the market". Though rational, agents do not hold the same amount of information. This is a major difference to the first model and to the EMH and is considerably closer to reality. Two types of investors are modelled: well-informed investors and poorly-informed investors, whose proportion is not known to market participants and especially to the market maker. Observing

the past actions of agents does not hence convey the same level of information anymore as it is not known whether the information contained in these actions is reliable – i.e. made by informed agents – or not – i.e. made by poorly informed agents. In this new model, where there is uncertainty about the value of the asset and about the composition of the market, Avery and Zemsky show by the means of simulations than bubbles may arise, as some agents may imitate the behaviour of past agents with the rational belief that these agents are well-informed.

This analysis is all the more so interesting when we consider it by looking at the current situation. The bubble and the following crash can in fact be thought of as the resulting phenomenon of a wrongly informed market by which I mean a market that mistakenly thought it was well-informed when it was not. As a consequence, and as described by Avery and Zemsky, a small event, that dents the confidence level of market participants or that generates reliable information to market participants, may generate a financial crash. Let us take for example, the ABX index that was launched in January 2007 in order to introduce some visibility and some transparency in a market – the market of CDS based on subprime home loans - where information was very poor. By aggregating some information that was until then unavailable to individual market participants, it brought some information to a market that was until then poorly-informed, and this event, as Avery and Zemsky call it, by bringing more reliable information to the market cast light on the fact that the market was previously poorly informed. The price had consequently to drop. It hence may have contributed to the crash of the subprime securities a couple of months later.

Considering this model where the composition of the market is key to account for market bubbles and market crashes, we have further tried to investigate the generation of bubbles by way of an informational approach. In this purpose, we considered a model that may account for rationally generated bubbles caused by the presence of even a very limited number of convinced investors – who strongly believe in a scenario – and a majority of rational investors, who rationally act according to the information gained from the previous actions of others agents. We strongly inspired from the work of Boot, Milbourn and Schmeits (2005) on credit ratings and multiple equilibria to derive our model.

III. Our model and results

Let there be one firm that is to invest in a project, for which it seeks \$1 of external financing. It can either invest in a viable (low-risk) project VP, or in a highly risky project HR. Both projects have NPVs such that NPV(VP) > NPV(HR) > 0. Project VP generates a cash-flow X_{VP} with probability p and 0 otherwise and project HR generates a cash-flow X_{HR} with probability q and 0 otherwise. We assume that $pX_{VP} > qX_{HR}$, i.e. project VP is first-best efficient.

The project in which the firm invests is private information of the firm. We assume that credit markets are perfectly competitive. Hence investors determine the repayment amount F that yields them zero expected profit. Therefore the repayment amount on the loan if investors anticipate that the firm chooses VP (and hence that the firm is of type G) is:

$$F_{VP} = \frac{1}{p}$$

Similarly the repayment amount on the loan if investors anticipate that the firm chooses HR (and hence that the firm is of type B) is:

$$F_{HR} = \frac{1}{q}$$

Let there be a proportion α of convinced investors in the market ($0 \leq \alpha \leq 1$) – who either believe that the firm invests in VP or in HR. The other (1- α) investors are not convinced at first and behave rationally.

Situation 1: All investors are rational (α=0)

We are exactly in the case of the general model described by Boot, Milbourn and Schmeits in the article *Credit Ratings as Coordination Mechanisms* (2005).

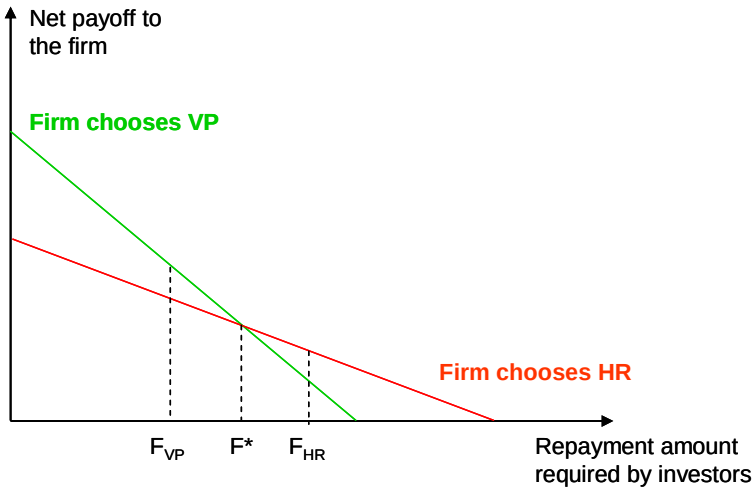
Let F_τ $(\tau \in \{VP, HR\})$ be the repayment amount required by investors. The net payoff to the firm if it chooses project VP is: $p(X_{VP}-F)$ The net payoff to the firm if it chooses project HR is: $q(X_{HR}-F)$

Let F* the repayment amount such that the firm is indifferent between VP and HR. F* is such that $p(X_{VP}-F^*) = q(X_{HR}-F^*)$, i.e.

$$F^* = \frac{pX_{VP} - qX_{HR}}{p-q}$$

$$F_{VP} \leq F^* \leq F_{HR}$$

We hence have, as pXVP > qXHR:



Graph 1

Result 1: The firm's project choice depends on investors' belief in the investment type of the firm.

The firm chooses the project that has been anticipated by rational investors. If investors believe that the firm will invest in the viable project, the firm optimally chooses to invest in

the viable project. Similarly, if investors anticipate project HR, then the firm will optimally choose to invest in HR.

This result shows us that in the case where all investors are rational, the firm's project choice in equilibrium is completely determined by rational investors' anticipations. The underlying logic is as follows:

- If investors believe the firm invests in project VP, they will require a repayment amount of F_{VP}. As $F_{VP} \leq F^*$ and net payoffs to the firm are strictly decreasing in F (whether if it chooses VP or HR), then we will have $p(X_{VP}-F_{VP}) \geq q(X_{HR}-F_{VP})$.
  The firm will hence rationally invest in project VP.

- If investors believe the firm invests in project HR, they will require a repayment amount of F_{HR}. As $F_{HR} \geq F^*$ and net payoffs to the firm are strictly decreasing in F, then we will have $q(X_{HR}-F_{HR}) \geq p(X_{VP}-F_{HR})$.
  The firm will hence rationally invest in project HR.

The beliefs of the rational investors are self-fulfilling. In fact in believing in a project choice, they will require a certain amount to be reimbursed which will drive the firm to choose the anticipated project at equilibrium. This self-fulfilling circle may make the firm optimally choose the highly risky project even if the viable project if first best efficient and easy to implement.

This result makes intuitively some sense as it appears that firms can no longer act only just as they like, but must take into account the opinions of numerous stakeholders. More particularly, most companies that decide to get public do so in order to get access to more funding sources (equity and debt). They can hence no longer ignore what the new fund providers believe and anticipate. It is nonetheless puzzling to think some of firms' key strategic decisions are only the result of exogenous agents. But what is interesting is to observe that the firm's fundamentals (growth strategy, market share, etc.) are not the only determinants of the firm behaviour – and in this model, they have no impact whatsoever.

Situation 2: All investors are convinced (α=1)

Let us now model a market exclusively composed of convinced investors. By convinced, we mean an investor who strongly believes that a firm chooses either the viable project (VP) or the highly-risky project (HR).

By a very similar reasoning, it can be shown that the self-fulfilling phenomenon describes above, also happens in the case where all investors are convinced. If investors are optimistic, i.e. believe that the firm will invest in VP, the firm will choose VP. Reciprocally if they anticipate HR, the firm will choose HR.

Again, this result can seem puzzling as it implies that even when convinced investors are wrong and believe that the firm will choose the highly risky project, it is nonetheless optimal for the firm to choose it.

Situation 3: There is a proportion α of convinced investors ($0 \leq \alpha \leq 1$) and (1-α) rational investors

We now generalize the model from Boot, Milbourn and Schmeits by modelling a market made of α% of convinced investors and (1- α)% of rational investors (for α any number between 0 and 1).

The (1- α)% of rational investors form rationally their belief about the firm's project choice and are aware of the presence of the α% of convinced investors. Let us assume that a proportion λ of convinced investors believe that the firm invests in VP ("the optimistics") and a proportion of 1- λ believe that the firm invests in HR ("the pessimistics"). This proportion is known to rational investors. The optimistics will hence require a repayment amount of F_{VP} and the pessimistics will require F_{HR}.

Let F_λ be the amount that rational investors require to be repaid to them.

We will show that if the proportion of convinced optimistic investors is large enough, the firm will optimally choose project VP and that if the proportion of convinced pessimistic investors is large enough, the firm will optimally choose project HR.

Result 2: In a situation where there is a proportion α of convinced investors – of which λ are optimistic and (1- λ) are pessimistic – and (1- α) rational investors, the firm's project choice at equilibrium depends on λ and is such that there exist proportions $\overline{\lambda}$ and $\underline{\lambda}$ such that:

- If $\lambda \leq \underline{\lambda}$, i.e. if convinced investors are pessimistic enough, then $F_\lambda = F_{HR}$ and the firm invests in HR.
- If $\lambda \geq \overline{\lambda}$, i.e. if convinced investors are optimistic enough, then $F_\lambda = F_{VP}$ and the firm invests in VP.
- If $\underline{\lambda} \leq \lambda \leq \overline{\lambda}$, then the firm's project choice depends on the proportion of optimistic and pessimistic investors in the market. It chooses whichever project the majority of convinced investors anticipate. If there is a proportion λ^+ of convinced investors, then $F_\lambda = F_{VP}$ and the firm invests in VP. If there is a proportion λ^- of convinced investors, then $F_\lambda = F_{HR}$ and the firm invests in HR.

(See proof in Appendix)

This result shows us that the firm's project choice at equilibrium entirely depends on the convictions of convinced investors, whatever their proportion in the market may be (α can be any number between 0 and 1). We find here again the self-fulfilling phenomenon in which the anticipations of a few people on the behaviour of a firm determine the very behaviour of this firm. Even a very small proportion of convinced investors $(\alpha \to 0)$ may drive the behaviour of the firm. We can furthermore derive from this result that a significant proportion of optimistic investors, i.e. anticipating project VP, can guide the firm behaviour to the best equilibrium, i.e. the one involving the first-best project VP. In fact, if convinced investors are optimistic, the amount required by all investors (convinced and rational) that the firm anticipates will be such that it will be optimally efficient for it to choose project VP.

This result is quite intuitive, as by considering the anticipations of a majority of convinced investors, one firm can anticipate what the repayment amount should be. What is less intuitive is to see that even when these convinced investors are in a very small minority, they can nonetheless determine the behaviour of the firm and of the remaining majority of investors.

Even if not intuitive at first, it can nonetheless be observed in practise that the beliefs of a few leaders may drive the behaviour of a majority (in but also out of the finance sphere). In the finance sphere more particularly, let us for example think of individuals who have such an influence on markets. Today Alan Greenspan can be seen as one of this kind of people, when you consider how one of his interventions may impact on the market (more specifically the interest rate market). Similarly it is not rare to observe that the investment recommendations of some specific research analysts may have considerable impact on the behaviour of investors (for example asset managers). The research analysts, who have such great influence, are often those who receive most media coverage and whose ex-ante forecasts are in average positively correlated with ex-post returns. More generally, convinced investors may be investors who have a higher level of private information than other market agents. From this perspective, institutional investors may be more likely to be "convinced" investors than individual ones as they can benefit from more information resources (databases, meetings with CFOs, etc.). Their behaviour may hence be considered as more reliable by other kind of investors who do not have higher information and who behave rationally. Let us for example think of how the actions of some private equity fund may influence the behaviour of some individual investors. Because they generally get significant returns from their LBOs, the PE funds to acquire even a minority stake in some companies may encourage individual investors to buy stocks of this company or to keep their stake in the company by not offering their stocks to the takeover bid. Exemplary enough, PAI never succeeded in acquiring more than 95% of Kaufman & Broad in 2007 (to benefit from the implied tax integration) because some minority investors refused to sell their stake in the company, forecasting that the changes to be implemented by PAI would mean high capital gains in the future. However, though convinced investors may be highly informed investors, we can nonetheless also figure that they may simply act on more irrational reasons, i.e. be "crazy" agents who just act on their inner conviction without relying on superior reliable private information. It may be worrying that "crazy" agents may completely drive the behaviour of the market, but it can nonetheless totally be conceived.

Result 3: The smaller the proportion of convinced agents, the higher their influence on the market.

This result may be intuitively understood. By driving the behaviour of (1-α) agents, α convinced investors influence the market. As α gets smaller, (1-α) gets larger and

consequently the influence of the convinced investors on the market gets bigger. This result may be intuitively understood in practise, when considering for example that convinced agents are highly informed investors. In fact it is generally observed that the intensity of the influence of convinced agents is stronger in a context where very little information is available or when uncertainty is high – in periods of crises for example. In periods of uncertainty, α should be very low, meaning that only a short minority of agents have a clear opinion on the situation. The other agents are rational and act according to their anticipations of what the convinced investors do. α is hence an indicator of the amount of uncertainty in the market. The smaller the α, the more uncertainty there is in the market. For a small α, the intensity of the influence will be very strong as convinced investors will be driving the behaviour of an important proportion (1- α) of rational investors.

Now that we described how the behaviour of rational markets may be influenced by the behaviour of some convinced investors, let us get back to our initial issue and try to understand how financial bubbles may be generated in rational markets, contrary to what the classical theory of efficient markets states. In this purpose let us get interested in the evolution of the equity value of the firm in our model. Intuitively the equity value should be highly dependant on the behaviour of convinced investors. In fact for their different convictions (optimistic/ pessimistic) they will drive different behaviours for the firm (invest in VP/ invest in HR). As a consequence, the payoffs received by the firm will be different if convinced investors are optimistic or if they are pessimistic, since the invested project will generate variable payoffs and the investors will require variable repayment amounts.

Result 4: In a situation where there is a proportion α of convinced investors – of which λ are optimistic and (1- λ) are pessimistic – and (1- α) rational investors, the firm's equity value at equilibrium depends on λ and is such that:

- If $\lambda \leq \underline{\lambda}$, investors require F_{HR} to be repaid and the firm chooses at equilibrium to invest in project HR. Hence the value of the firm is $E(\lambda) = q(X_{HR}-F_{HR})$.
- If $\lambda \geq \overline{\lambda}$, investors require F_{VP} to be repaid and the firm chooses at equilibrium to invest in project VP. Hence the value of the firm is $E(\lambda) = p(X_{VP}-F_{VP})$.

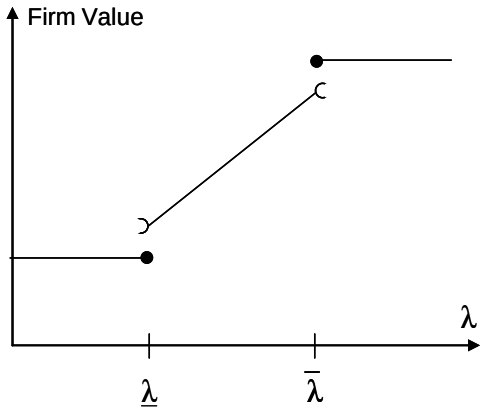
- If $\underline{\lambda} \le \lambda \le \overline{\lambda}$, the value of the firm depends on the proportion of optimistic and pessimistic investors and is such that:
  $E(\lambda) = \lambda p(X_{VP} - F_{VP}) + (1-\lambda)q(X_{HR} - F_{HR})$ which is increasing in λ.

Furthermore, we have, for λ such that $\underline{\lambda} < \lambda < \overline{\lambda}$:

$$E(\underline{\lambda}) = \underline{\lambda} p(X_{VP} - F_{VP}) + (1 - \underline{\lambda}) q(X_{HR} - F_{HR}) > q(X_{HR} - F_{HR}) \text{ and}$$

$$E(\overline{\lambda}) = \overline{\lambda} p(X_{VP} - F_{VP}) + (1 - \overline{\lambda}) q(X_{HR} - F_{HR}) < p(X_{VP} - F_{VP})$$

The firm value is hence such that:



Graph 2

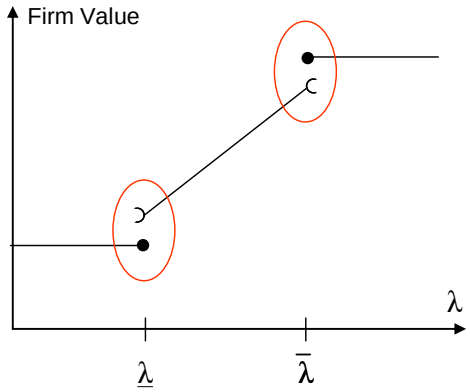
From this result we can derive that when convinced investors are optimistic enough ($\lambda \ge \overline{\lambda}$) the firm's equity value (and hence stock price) is high. In fact when convinced investors are optimistic enough, the firm invests in VP and all investors (convinced and rational investors) require a low repayment amount of F_{VP} (Result 2). The firm hence gets a high payoff of X_{VP} with probability p from the project and has to repay only F_{VP}; its equity value is hence high and equal to $p(X_{VP}-F_{VP})$. Similarly, when investors are pessimistic enough ($\lambda \le \underline{\lambda}$), the firm's

equity value is low and equal to $q(X_{HR}-F_{HR})$. In the intermediate zone ($\underline{\lambda} \leq \lambda \leq \overline{\lambda}$), the value of the firm's equity is dependent on the proportion of optimistic and pessimistic agents. The firm's equity value is intuitively increasing with λ.

Furthermore, we can note that for every λ, there is a single equity value for the firm and vice-versa. As a consequence, by observing a price on the market, rational investors can deduce the proportion λ of the convinced investors who are optimistic and from this anticipate the behaviour of the firm and require a certain repayment amount. We can similarly consider the situation on a dynamic point of view to account for changes in stock prices. Let us for example assume that the stock price of the company is somewhere between $q(X_{HR}-F_{HR})$ and $p(X_{VP}-F_{VP})$. A rational investor would then deduce that the level of optimistic investors is somewhere between $\underline{\lambda}$ and $\overline{\lambda}$. Let us now assume that the stock price suddenly increases or that a flow of buy orders reaches the market. The rational investor could hence deduce that he underestimated the level of λ, would increase his level of λ, which could change – assuming all rational investors have the same reaction – the situation at equilibrium and may increase the firm's equity value (cf. graph 2). Observing an increase in prices may hence drive upward moves in the stock price and generate a bullish financial bubble, as more buy orders are observed and as rational investors consequently reassess their estimates of λ upwards. Similarly, by observing decreases in the stock price or a flow of sell orders; the rational investor may deduce he overestimated the proportion of optimistic investors, reduce his level of λ and drive downward moves in the stock price, generating a downward financial trend.

It is interesting to note that the firm's equity value can go from very stable states to very volatile ones. In fact when $\lambda \leq \underline{\lambda}$, the firm's equity value is stable and equal to $q(X_{HR}-F_{HR})$ whichever the proportion λ may be. When $\lambda \geq \overline{\lambda}$ similarly, the firm's equity value also remains stable and equal to $p(X_{VP}-F_{VP})$. However as soon as $\underline{\lambda} \leq \lambda \leq \overline{\lambda}$, the equity value gets highly dependant on the level of λ, as was previously observed. The stock price gets hence very volatile and may vary significantly from one value of λ to another. By reassessing their estimates of λ downwards – as per the mechanism described above - rational investors may hence cause very rapid decreases to stock prices as soon as their estimate of λ gets lower than $\overline{\lambda}$.

It is furthermore interesting to note that there is a significant drop in the equity value of the firm as soon as λ gets lower than $\overline{\lambda}$ and that there is an important rise in the price as soon as λ gets higher than $\underline{\lambda}$. The volatility of the stock price is hence very high in these areas and may signal the beginning of a period of instability in the market. Interestingly enough, these areas of large instability precede periods of drops or increases in prices.



By reappraising the level of optimistic investors in the market, investors may hence cause very rapid and sudden increases or drops in share prices. Rational bubbles/ crashes may hence be generated as stock prices may experience dramatic and instable increases/ decreases that are not connected to any change in the company's fundamentals (which remain the same).

To sum up, we showed that the behaviour of even a few convinced investors may drive the behaviour of a rational market and determine stock prices. When the market is certain that there are enough optimistic (resp. pessimistic) investors in the market, the stock price remains stable and high (resp. low). However, rational bubbles may be generated, as rational investors reassess their estimates about the opinions of the convinced agents, causing rapid rises (resp. drops) in stock prices in a context of high volatility.

These results remain however theoretical. It may hence be interesting to think about ways to empirically demonstrate this model or at least think about empirical elements that could be in line with these results.

Validating our model empirically

Studying the correlations between the opinions of convinced agents and the ex post behaviours of the firm

We first showed with our model that the behaviours of a couple of convinced investors may drive the behaviour of all the rational market and the behaviour of the firms that need to get access to the bond market to finance their development (cf. Result 2). What would be interesting would consequently be to look at a possible correlation between the announcements of certain convinced investors and the ex-post investment decisions of companies. The objective of this empirical study would be to check to what extent they do follow what the market thinks and forecasts. For example, if some research analyst is very well-regarded in his sector, it would be interesting to check if a large cap of this sector is following some of the recommendations the analyst publishes in his research papers. More specifically if the research analyst was forecasting that it would be interesting for the firm to develop in a certain market, on a certain segment or by merging with the company X, it would be very interesting to study to what extent these recommendations or opinions of the analyst are indeed followed by the company.

A positive correlation would mean that the company indeed follows some of the recommendations of the analyst. We should expect to observe in practise such positive correlations in average as larges caps that are traded on stock exchanges and get some of their funding on the bond market can no longer act without taking the opinions of the market into account. More interesting for us would be to check if these correlations are close to 1. This would mean not only that firms' behaviours are indeed inspired by the opinions of some convinced agents, but more interestingly that the latter are determinant for firms' behaviours and strategic decisions.

In practise it may be useful to consider a sample of large caps (more interesting that mid or small cap as large caps tend to be more in need of the market than mid or small caps are) belonging to a geographic area or belonging to a specific sector and to study the correlations of their strategic decisions with the ex-ante opinions of some convinced agents. The trickiest part would undoubtedly be to identify a convinced agent who would be able to have some significant influence on his sector (on the investors and on the companies). That is why the

first step of the study would certainly be to identify such research analyst and only then, to build a sample of large caps acting in the coverage field of the research analyst.

We considered as an example of convinced agent a research analyst. But as we have seen previously, convinced agents may be from a complete another nature. Similar empirical studies may be imagined to consider different kinds of "convinced" investors.

Studying the correlations between the evolution in the stock price of a company and the debt interests of its corporate bonds

In to our model we showed that when convinced agents were optimistic enough, the company invested in the viable project, the bond investors required a low repayment amount and the stock price of the company was high (cf. graph 2). Similarly when convinced agents were pessimistic enough, i.e. believed that the company would invest in the risky project, the company invested in the risky project indeed, the bond investors required a high repayment amount and the stock price of the company got low. We hence showed that there was a positive correlation between the stock price of a company and the debt repayment amount required by bond investors: high stock prices seem to be connected to low repayment amounts and low stock prices seem to be connected to high repayment amount. It would hence be interesting to study if this result is observed in practise and if there exists indeed a positive correlation between stock prices and the repayment amounts required by the corporate bondholders.

To do so it would be relevant to consider a sample of large caps (for the same reasons than previously explained) that are active on the corporate bond market to assess the correlations between the stock price of the company when the bond is issued and the repayment amount bondholders require. It may be even more interesting to consider, for a given company, what the correlations were for different bond issue in time. This could indeed give us a dynamic point of view on how the stock price of the company and the bond repayment amounts evolve together.

Intuitively these two components should be positively correlated in practise. In fact, companies with low stock prices, having a low equity value, should be more likely to default

than similar companies with high equity value. The risk is hence significantly higher and the repayment amount required by bondholders should hence be significantly higher in order to make up for the additional risk taken. Reciprocally when bondholders require high repayment amounts, the debt structure should appear to be riskier, generally because the debt amount is higher. As the firm value is independent from the financing structure, the equity value should be consequently lower for higher debt amounts. As a consequence, the stock price should be low.

The positive correlation between the stock price of the company and the repayment amounts required by bondholders seems indeed to be quite intuitive. It may nonetheless be interesting to observe it empirically so as to give more weight to our model.

Studying the correlations between the volatility of stock prices and the beginning of financial crashes

As we have previously seen (more particularly see graph 2), as soon as the stock price decreases from its high stable level, it volatility is significantly increased. The volatility is especially high when the proportion of optimistic agents gets lower than the upper threshold $\bar{\lambda}$. That means that when the stock price suddenly drops from a high stable level and that a financial crash may start – because rational investors reassess their proportion of optimistic agents in the market downwards – the volatility of the stock price significantly increases and is particularly high at the very beginning of the crash. Can this high volatility be observed in practise? In this regard it would be interesting to look at past financial crashes and to assess each time if the volatility of stock prices is especially high before the crash actually happens.

If we look at the current crisis, it may be interesting to note that the volatility of markets had already increased before the crisis actually burst out. The VIX index for example, measuring the volatility of the American market, started to double in a couple of days at the beginning of 2006, i.e. far before the subprime crisis burst out (summer 2006). This change was all the more noticeable that the previous period had been characterised by very low volatility on markets.

Hence, even if it is more commonly known that the volatility of asset prices may increase in periods of crises, it is interesting to note that volatility seems to increase even before the crisis actually emerges. Testing empirically this observation using different crises would be very useful to check this hypothesis.

Conclusion

Through the establishment of a simple model for investors' behaviours on markets we have shown that financial bubbles may be rationally generated in spite of the idea, commonly spread by press or public opinion, that financial bubbles offer the proof that financial markets are led by irrational agents. We have hence found the same results than a large part of the existing research that has had an informational approach to deal with the problem of rational bubbles had found before, but by taking another perspective. In fact though most of the existing research focused their model on the fundamental value of the asset, we chose to focus our model on the psychology of investors. More concretely whereas the market information that reaches the market agents deals with the value of the asset in most research, it deals with the psychology of the investors in our model – more specifically the proportion of optimistic and pessimistic investors among convinced agents.

We have shown in our model that even a minority of convinced agents may fully drive the behaviour of a rational market, i.e. of its investors and of its issuers (companies that need to issue debt securities to finance their activities). The valuation of a firm consequently depends on what the investors think the psychology of the convinced investors is and may significantly move as rational investors reassess their beliefs on the behaviour of the convinced investors. We have hence shown that financial bubbles may be rationally generated and that they are in someway intrinsic to the system as they result from the rational behaviour of investors who optimally behave.

As a matter to further study, it would be interesting to deepen our analysis and make it more valuable by completing it by an empirical approach – as our model remains for the moment purely theoretical. Some empirical studies, like for example studying correlations between the price of an asset and the repayment amounts required on its corporate bonds, could be considered in order to give more weight to our findings. Another angle of study could be to adopt a probabilistic approach, as has been more generally done in research (see Bikhchandani, Hishleifer and Welch for example) and, by considering the same market composition – convinced investors (optimistic and pessimistic) and rational investors – understand how the prices are set and see if similar rational bubbles may be generated.

Considered under the current circumstances, it is interesting to note how our model reasserts the importance of the notion of confidence on markets (see the crucial importance of the proportion of optimistic investors in our model). Interestingly enough – and that could be a positive note deriving from our model in the present times – it seems that in order to get out of the financial crash, we would only need to persuade the minority of convinced agents that the situation should get much better to get the whole market to rise again (due to the self-fulfilling effect we described ion our model). Though hopeful at the moment, this effect still needs however to be observed in practise.

Appendix

Proof of result 2

The expected payoff for rational investors is $-1 + \lambda p F_\lambda + (1-\lambda) q F_\lambda$, which, assuming perfect competition, must be nil. Hence,

$$F_\lambda = \frac{1}{\lambda p + (1-\lambda) q}$$

Let us note that $F_{VP} \leq F_\lambda \leq F_{HR}$.

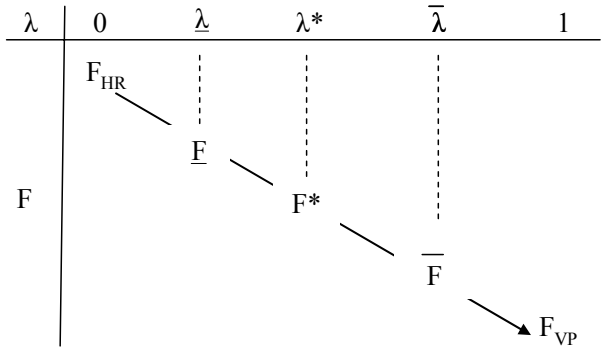
$F_0 = F_{HR}$ and $F_1 = F_{VP}$.
As $F_{VP} \leq F^* \leq F_{HR}$ and F_λ is continuous and strictly decreasing in λ, $\exists! \lambda^* \in [0,1] / F_{\lambda^*} = F^*$

The firm's expected amount to be repaid to investors is:
$F(\lambda)=\alpha[\lambda F_{VP} + (1- \lambda)F_{HR}] + (1-\alpha) F_\lambda$

As $F'(\lambda) = \alpha\left(\frac{1}{p} - \frac{1}{q}\right) - (1-\alpha)\frac{\lambda(p-q)}{[\lambda p + (1-\lambda)q]^2} < 0$

F is strictly decreasing in λ and as $F^* \in [F_{VP}, F_{HR}]$, $\exists! \lambda^* \in [0,1] / F(\lambda^*) = F^*$

Let $\underline{\lambda} \in [0, \lambda^*]$. $F(\underline{\lambda}) = \underline{F}$
Let $\overline{\lambda} \in [\lambda^*, 1]$. $F(\overline{\lambda}) = \overline{F}$



For any $\lambda \le \underline{\lambda}$, $F(\lambda) \in [\underline{F}, F_{HR}]$. Hence $q(X_{HR}-F(\lambda)) \ge p(X_{VP}-F(\lambda))$ (see graph 1) and the firm optimally chooses to invest in HR. Rational investors anticipate this and hence require $F_\lambda=F_{HR}$.

For any $\lambda \ge \overline{\lambda}$, $F(\lambda) \in [F_{VP}, \overline{F}]$. Hence $p(X_{VP}-F(\lambda)) \ge q(X_{HR}-F(\lambda))$ (see graph 1) and the firm optimally chooses to invest in VP. Rational investors anticipate this and hence require $F_\lambda=F_{VP}$.

For any $\lambda \in \left[\underline{\lambda}, \overline{\lambda}\right]$, $F(\lambda) \in [\overline{F}, \underline{F}]$. Hence

- If $\lambda \ge \lambda^*$ (let's call it λ^+), $p(X_{VP}-F(\lambda)) \ge q(X_{HR}-F(\lambda))$ and the firm optimally invests in VP. Rational investors anticipate this and hence require $F_\lambda=F_{VP}$.

- If $\lambda \le \lambda^*$ (let's call it λ^-), $q(X_{HR}-F(\lambda)) \ge p(X_{VP}-F(\lambda))$ and the firm optimally invests in HR. Rational investors anticipate this and hence require $F_\lambda=F_{HR}$.

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