Continuous DCF Method

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1 Introduction

The discounted cash flow method (DCF) is a method of valuing a company based on the time value of its future cash flows. This is a famous technique employed very frequently in investment banking. It is probably the best way to estimate a company on the basis of its future path and not on the basis of its past behaviour or its competitor's value. However, this method contains two major drawbacks.

Firstly, the result suffers from a clear lack of accuracy. This is due to the inherent uncertainty of the forecasts. For instance, the market growth, the rise of a competitor, the economic conditions are macroeconomic or microeconomic factors that can hardly be estimated precisely. We could argue that the enterprise value obtained contains an margin error of $\pm 15\%$.

Secondly, due to the high number of parameters involved, this method is time consuming. The business plan needs to be drawn properly and prolonged until a defined business horizon.

The main idea of this work is to suggest modestly a method requiring less parameters, and therefore faster, without losing too much on the accuracy. In other words, we want to reduce the second drawback without increasing the first one.

In that perspective, we determine the parameters weighing the most in the valuation, and we put ourselves under a continuous time in order to take advantage of the integration theory.

First of all, we set forth a graphical approach of the DCF method and we present it as a result coming from a system of two equations.

After that, we select the main parameters and model the EBIT dynamic continuously. Then, we find the properties and characteristics of our continuous DCF valuation under some assumptions.

In the last part we present some examples and we estimate the accuracy of this model, especially with the statistical test of Student.

2 A graphical approach of the DCF Method

As explained in the introduction, the DCF method values a company with the NPV of its future cash flows. Following a basic intuition, we could think that focusing on the cash flows is more important than focusing on the discount rate. This is wrong : equal attention should be paid to the free cash flows and to the discount factor, as we shall demonstrate.

The first equation that would come to the mind of someone describing the DCF is probably the following one :

$$EV = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t}$$

Where EV is the Enterprise Value and t the index of a future year. We assume that the WACC is constant over time.

Then a definition of the WACC would come. The Weighted Average Cost Of Capital is the rate at which a company is expected to finance itself through equity and debt. It is described by the following formula :

$$WACC = \frac{k_e EqV + (1-\tau)k_d ND}{EV}$$

Where EqV is the Equity Value, ND is the Net Debt, k_e is the cost of equity, k_d is the cost of debt and τ is the tax rate. This equation computes the NPV of the future free cash flows.

Very often, the description of the DCF ends there, without pointing out the crucial link between those two equations. If we use the fact that the Enterprise Value is the sum of the Equity Value and the Net Debt, $EV = EqV + ND$ we observe that the second equation becomes :

$$WACC = \frac{k_e(EV - ND) + (1-\tau)k_d(ND)}{EV}$$

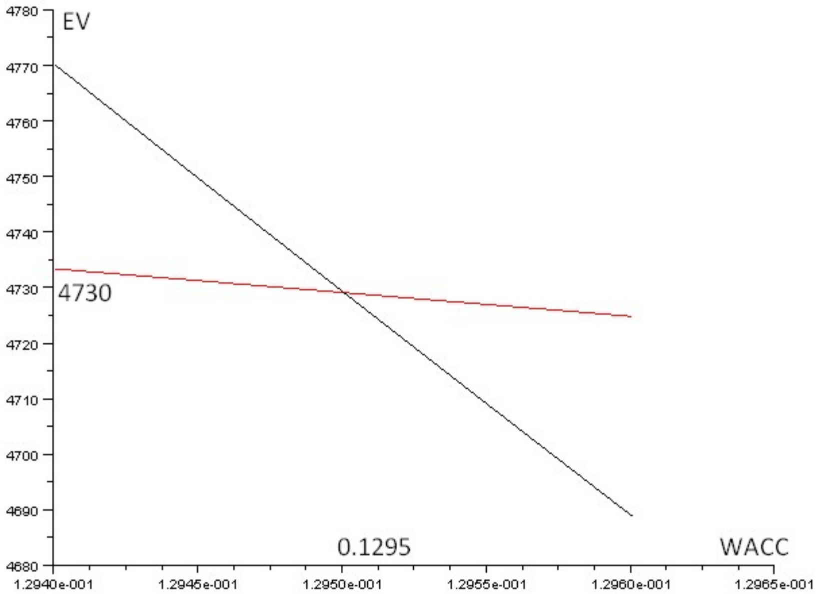
This leads to :

$$EV = ND\frac{k_d(1-\tau) - k_e}{WACC - k_e}$$

This quick operation shows that the role of the second equation goes further than only computing the WACC. We end up with a system of two equations with two unknowns : the Enterprise Value and the WACC.

$$\begin{cases} EV = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t} \\ \\ EV = ND\frac{k_d(1-\tau)-k_e}{WACC-k_e} \end{cases}$$

Graphically, this means that the Enterprise Value and the WACC are the result of an intersection of 2 curves. For instance, in the example below we find an EV of 4730 and a WACC of 12.9% .



Unfortunately this system is too complicated to obtain an independent expression for each unknown. To overcome this difficulty and to grab the solution, the analyst will create a loop on its speadsheet. In other words, he creates an iterated sequence to converge toward the intersection, an attractive fixed point.

Throughout the following pages, we aim at simplifying the computation of the sum of the Free Cash Flows. The goal is to derive an equation clear enough to draw its curve properly and promptly. Since the second equation does not change, we only touch to half of the problem. This allows us to expect a good accuracy.

3 A quick approximation of the discrete DCF valuation

In the previous section, we have seen that the pair (EV, WACC) is defined by the following system :

$$\begin{cases} EV = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t} \\ \\ EV = ND\frac{k_d(1-\tau)-k_e}{WACC-k_e} \end{cases}$$

Let's focus ourselves on the first equation and let's try to get a first approximation of the inifite sum.
The Free Cash Flows (FCF) are computed as follows :

$$FCF = EBIT + D\&A - Taxes - CAPEX - \Delta WCR$$

We assume that :

1. The EBIT grows at a constant rate g.
2. The D& A and the CAPEX compensate each other. This may be true for any company on the long run.
3. The change in working capital (ΔWCR) is small enough to be included in our margin of error.
4. The WACC is constant over time.

Let us define $EBIT_0$ as the initial EBIT.

That way we obtain the following dynamic for the free cash flows :

$$FCF_t = (1-\tau)EBIT_0(1+g)^t$$

and then it leads us to :

$$EV = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t} = \sum_{t=1}^{\infty} \frac{(1-\tau)EBIT_0(1+g)^t}{(1+WACC)^t}$$

We compute the infinite sum assuming that $g < k$ and finally derive the formula we were looking for :

$$EV = (1-\tau)\frac{EBIT_0(1+g)}{WACC-g}$$

As expected, the enterprise value (EV) increases with the inital level of EBIT and with the growth rate. On the other hand, the tax rate τ and the WACC k have a negative impact.
The system giving the solution fo the pair (EV, WACC) then becomes :

$$\begin{cases} EV = (1-\tau)\frac{EBIT_0}{WACC-g} \\ \\ EV = ND\frac{k_d(1-\tau)-k_e}{WACC-k_e} \end{cases}$$

This time, this system gives an independant solution for each unknown :

$$\begin{cases} EV = (1-\tau)(\frac{1}{X}-1)\frac{EBIT_0}{g-k_e} \\ \\ WACC = \frac{g-k_e X}{1-X} \end{cases}$$

Where $X = \frac{(1-\tau)EBIT_0}{ND(k_d(1-\tau)-k_e)}$

It could be interesting to study the clear impact of each parameter on the Enterprise Value and the WACC, with the computation of the sensibilities for instance. However, this model is quite too simple to be use in practice since the shape of the $EBIT$ (an increase of $g\%$ each year) cannot always reflect the EBIT dynamic of a random company. That way we prefer to design a quite more complicated model with less restrictive assumptions.

4 The continuous approach

Let us now switch to a continuous approach of the DCF. All the different variables (Revenues, EBIT, D& A etc...) are not considered discrete anymore but continuous. This will allow us to benefit from the power of the integration theory. Since the time is now continuous, discount factors are now exponential.

4.1 Modelling the EBIT curve

As we have seen in part 3, the EBIT takes a significant part in the estimation of the Free Cash Flows. This is partly due to the fact that the Capex and the D& A almost compensate themselves on the long run in many cases.

Except in some very volatile business plans, the EBIT evolution can be well approximated by 4 parameters :

- its initial level $EBIT_0$
- its initial growth rate g_0
- its final growth rate g_∞
- the period of transition between those two growth rates

That way, we shape the EBIT dynamic as follows :

$$EBIT_t = EBIT_0(e^{-\lambda t}e^{g_0 t} + (1 - e^{-\lambda t})e^{g_\infty t})$$

Basically, we have an exponential transition from the initial growth g_0 to the infinite growth g_∞. This transition is defined by a half-life period $T_{1/2} = \frac{\log 2}{\lambda}$. In other words, after a period of $T_{1/2}$ we know that half of the transition has been completed. It means that our exponential coefficient $e^{-\lambda t}$, which is equal to 1 when $t = 0$, is equal to 1/2 when $t = T_{1/2}$. The formula given previously comes from the equation $\frac{1}{2} = e^{-\lambda T_{1/2}}$.

More precisely, when t is low compared to the transition period $T_{1/2}$, that is to say when t is near zero, we have a growth rate of g_0 :

$$EBIT_t \sim EBIT_0(e^{g_0 t})$$

On the other hand, when t is significant compared to the transition period $T_{1/2}$, that is to say when t is high, we have a growth rate of g_∞ :

$$EBIT_t \sim EBIT_0(e^{g_\infty t})$$

Between those two extreme cases, the growth rate is a weighted average of those two growth rates g_0 and g_∞. The weight on the growth rate of g_0 is : $e^{-\lambda t}$. It is equal to 1 when t is low and it is equal to zero when t is high. At the same time, the weight on the growth rate of g_∞ is : $1 - e^{-\lambda t}$. It is the exact contrary of the pevious one. This weight is equal to 0 when t is low and equal to 1 when t is high. The sum of those two weights is of course equal to 1.

As said before, λ is the exponential factor related to the period of the transition between g_0 and g_∞. Since it is an exponential decay, the half life period of transition is $T_{1/2} = \frac{\log 2}{\lambda}$.

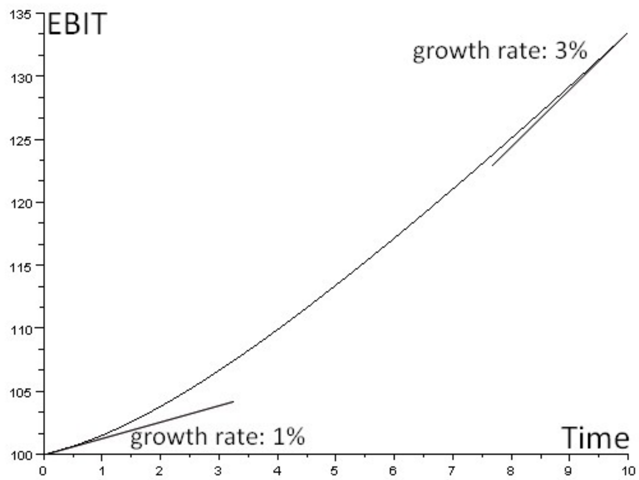
As usual, the transition is 94% achieved when $T = 4T_{1/2}$. For this reason, the relationship between λ and the period of transition T is :

$$\lambda = \frac{4 \log 2}{T}$$

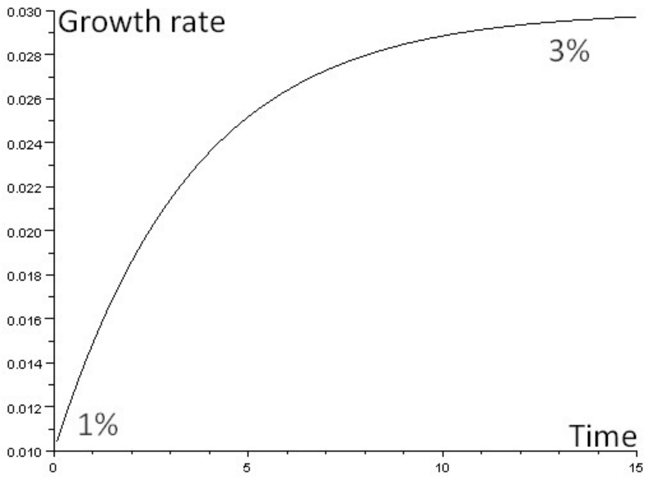
We have chosen an exponential decay since is a quite smooth transition where the period of transition is easy to adjust. On top of that, this expression will be easy to integrate in the future.

We could have chosen a dynamic on g itself, obtaining something not to far from : $EBIT_t = EBIT_0(e^{g(t)t})$. However, this kind of formula would be very hard to integrate, even for a linear dynamic of g_t.

See below an example of an EBIT curve (given by $EBIT_0 = 100$, $g_0 = 0.01$, $g_\infty = 0.03$, $T = 10$years)



The shape of its instant growth rate is given below.



We can see that the transition between the g_0 growth rate (1%) and the g_∞ infinite growth rate (3%) is smooth.

We will see in the part 5 how the g_0, g_∞ and T parameters are computed.

4.2 Main assumptions

We make the following assumptions :

1. The EBIT grows at a constant rate g_0 at the begining and at a rate g_∞ on the long run.
2. The D& A and the CAPEX compensate each other. This may be true for any company on the long run.
3. The change in working capital (ΔWCR) is small enough to be included in our margin of error.
4. The WACC is constant over time.

The FCF over time is given by :

$$FCF_t = (1-\tau)EBIT_t$$

In our continuous world, the formula giving the Enterprise Value is the following one :

$$EV = \int_0^\infty e^{-kt} FCF_t dt$$

Where k is the continuous weighted average cost of capital (WACC) linked to the traditional WACC by : $k = \log(1 + WACC)$.
By using the assumptions 2 and 3 we obtain the following formula for the Enterprise Value :

$$EV = \int_0^\infty e^{-kt}(1-\tau)EBIT_t dt$$

We use the first assumption and the EBIT dynamic chosen previously to compute the integral.

$$EV = \int_0^\infty e^{-kt}(1-\tau)\Big(EBIT_0(e^{-\lambda t}e^{g_0 t} + (1-e^{-\lambda t})e^{g_\infty t})\Big)dt$$

$$EV = (1-\tau)EBIT_0(\frac{1}{k-g_\infty} + \frac{1}{k+\lambda-g_0} - \frac{1}{k+\lambda-g_\infty})$$

We have obtained a simpler first equation of the system giving the pair (EV, WACC). The system is now :

$$\begin{cases} EV = (1-\tau)EBIT_0(\frac{1}{k-g_\infty} + \frac{1}{k+\lambda-g_0} - \frac{1}{k+\lambda-g_\infty}) \\ \\ EV = ND\frac{k_d(1-\tau)-k_e}{WACC-k_e} \end{cases}$$

We notice that if $g_0 = g_\infty$ we obtain the formula derived in the part 3. Some adjustments are made in the following example to take into account the change in Working Capital Requirement (ΔWCR).

4.3 Change in working capital requirement

If the business Plan shows us that the Change in Working Capital Requirement (ΔWCR) is sufficiently different from 0, we add a last term in our formula. This last term corresponds to the sum of a change in Working Capital Requirement growing at rate g_∞. Taking the infinite growth rate is relevant since the working capital is often assumed to be proportional to the sales, and then governed on the long run by the same growth rate than the sales, the EBITDA and the EBIT.

$$\sum^{\infty} \Delta WCR_t = \frac{\Delta WCR_\infty}{k - g_\infty}$$

Where ΔWCR_∞ is a well chosen change of the WCR on the long run. This value is almost constant at the end of the business horizon.

The final formula for the Enterprise Value becomes :

$$EV = (1-\tau)EBIT_0(\frac{1}{k-g_\infty} + \frac{1}{k+\lambda-g_0} - \frac{1}{k+\lambda-g_\infty}) - \frac{\Delta WCR_\infty}{k-g_\infty}$$

This is the formula we will use after to estimate the accuracy of this model. The final system is then :

$$\begin{cases} EV = (1-\tau)EBIT_0(\frac{1}{k-g_\infty} + \frac{1}{k+\lambda-g_0} - \frac{1}{k+\lambda-g_\infty}) - \frac{\Delta WCR_\infty}{k-g_\infty} \\ \\ EV = ND\frac{k_d(1-\tau)-k_e}{WACC-k_e} \end{cases}$$

We keep in mind that k is the continuous WACC, and that k is related to the usual WACC by the following relationship : $k = \log(1 + WACC)$.

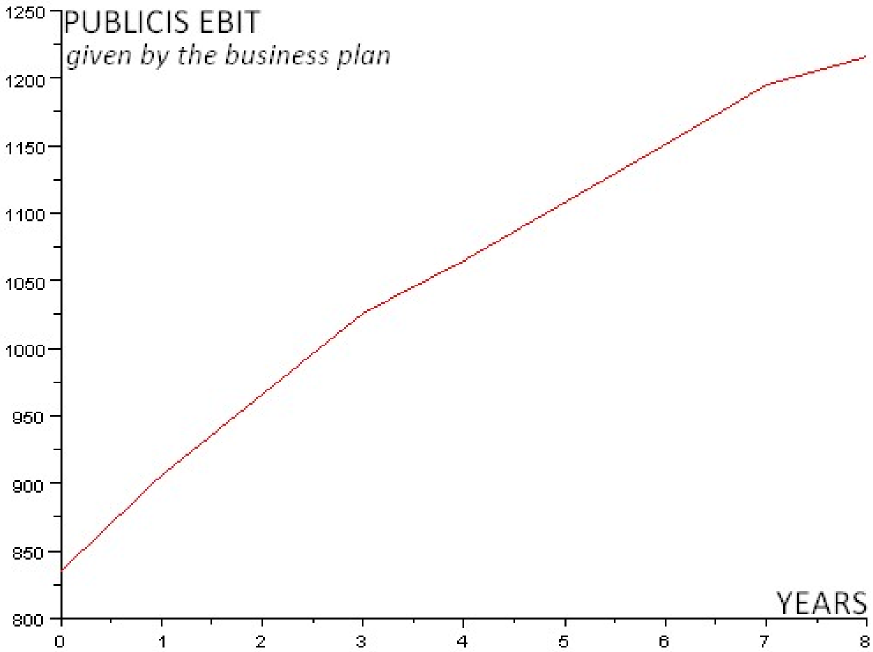
5 Calibration and use of the Model

5.1 Calibration

The parameter T reflects the duration of the transition between the initial EBIT growth g_0 and the perpetual EBIT growth g_∞. Since the EBIT has a significant weight in the valuation, finding a good value for T is fundamental.

By definition T should have the same order of magnitude than the business plan. The shorter is the transition, the lower is T.

To get the best value for T we calibrate the EBIT curve. First of all, we draw the expected EBIT of the following years. This is given by the business plan and we derive a curve like the one below :



After that, we take into account the initial growth g_0 and the perpetual growth g_∞, provided by the business plan as well. We obtain in our example :

$$g_0 = \ln(\frac{\text{EBIT}_1}{\text{EBIT}_0}) \sim \frac{\text{EBIT}_1}{\text{EBIT}_0} - 1 = 8.5\%$$

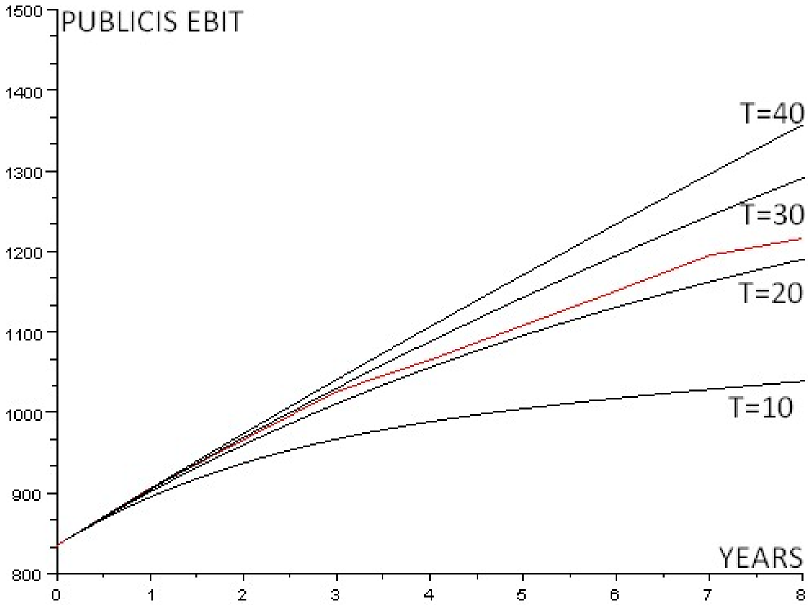
And g_∞ is the perpetuity growth rate : $g_\infty = 1.8\%$.

The last thing to do is to choose T such that the model curve fit as well as possible the expected EBIT curve. This could be done thanks to some mathematical methods by solving the following optimization problem :

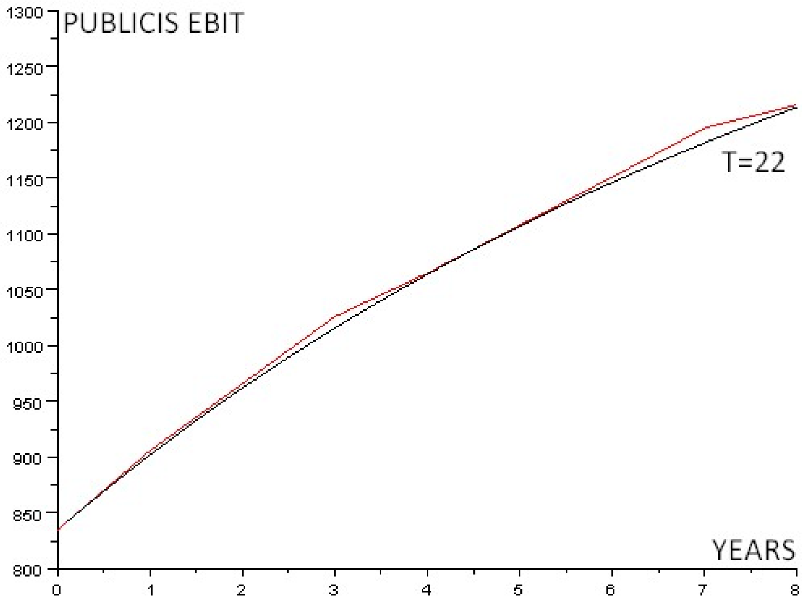
$$T = \underset{T \in \mathbb{R}^+}{\text{argmin}} \int_{t=0}^{\infty} \parallel \text{EBIT}_t^{real} - \text{EBIT}_t^{model}(g_0, g_\infty, T) \parallel dt$$

However, we prefer to do it manually. We will see later that this precision is good enough and that T can be taken as an integer value. To do this, we draw the EBIT curve for different T values and we pick up the one that does better match the real EBIT curve from the business plan.

For instance, see below the model curve depending on different values for T.



Finally, we notice that the best value for T is : $T = 22$. It gives us the black curve below :



Link with the business plan horizon
According to what we know about the exponential decay, T is not supposed to be equal to the Business Plan horizon, let's say 10 years (we assume here that the horizon corresponds to the transition period which is not always the case). However, $T_{1/2}$ is supposed to be approximatively equal to the the business plan horizon, since it the time when half of the transition is achieved. We obtain the value of T with the relationship : $T = 4T_{1/2}$.

In our example, the transition is half completed after 5 years. It gives us $T = 4T_{1/2} = 20$. However this handmade method is far less accurate than the one with the calibration.

Reminder
It is important to keep in mind that the transition is exponential and may drop quite quickly at the beginning. In other words, the transition is not linear and goes faster in the first years.

What is more, the discount factor increases as time goes by. For this reason, we should pay a little bit more attention to the way the Model curve fits the real one in the first few years than the way it fits it a decade after. On the other hand, it is also important to match the level of EBIT at the end of the business horizon to obtain an acceptable terminal value.

5.2 Use of the model

Here we sum up how the model should be used.

1) Collect the data
First of all we need to gather all the required data. It includes the expected EBIT, the tax rate, the change in working capital, the net debt, the cost of equity and the cost of debt. The four first items should be in the business plan. The cost of equity and the cost of debt can be computed or they should be included in a financial report or in a borker note.

2) Compute the growth rates
Then we need to compute the growth rates g_0 and g_∞ with the following expressions :

$$g_0 = log(\frac{\text{EBIT}_1}{\text{EBIT}_0}) \sim \frac{\text{EBIT}_1}{\text{EBIT}_0} - 1$$

$$g_\infty = log(1 + g_{perpetualBP}) \sim g_{perpetualBP}$$

3) Calibration
As explained before, the value of T needs to be computed thanks to the manual calibration of the EBIT curve. The T has to minimize the distance between the expected curve (from the Business Plan) and the Model curve.

4) Solving the equations
Once we have found all the parameters thanks to the previous steps, we can solve the system of two equations :

$$\begin{cases} EV = (1-\tau)EBIT_0(\frac{1}{k-g_\infty} + \frac{1}{k+\lambda-g_0} - \frac{1}{k+\lambda-g_\infty}) - \frac{\Delta WCR_\infty}{k-g_\infty} \\ \\ EV = ND\frac{k_d(1-\tau)-k_e}{WACC-k_e} \end{cases}$$

The unknows are the Enterprise Value (EV) and the WACC. Graphically it is the intersection between two curves. Obtaining the right values can be done graphically or easily with the Newton-Raphson method.

As usual, the Equity Value ot the company is obtained by subtracting the net debt to the enterprise value ($EqV = EV - ND$). We notice that in absolute terms, the error of this model on the Equity Value is the same than on the Enterprise Value. This comes from the fact that we do not modify the net debt.

6 Example : PUBLICIS

In this section we compare the results given by a usual DCF and the results given by the continuous DCF for Publicis. We will focus on the accuracy of the Enterprise value and on the accuracy of the WACC. However, it is important to take into account that the second method requires less time and parameters.

6.1 Usual DCF

Risk free rate	3,50%
Beta (Datastream)	-
Unleveraged beta	1,1
Leveraged beta	1,2
Market risk premium	7,50%
Cost of equity (k)	**12,5%**
Beta of the debt	-
Cost of net debt	
Pretax (based on CAPM)	3,18%
Post tax	**2,13%**
WACC (K)	**12,13%**

Sum of discounted FCF	2 674
Terminal Value	3 203
Enterprise Value	**5 877**
Last net debt	210
Equity Value	**5 667**

Perpetuity growth rate	**2,5%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**906**	**966**	**1 026**	**1 065**	**1 108**	**1 151**	**1 195**	**1 216**
(Corporate tax on EBIT)		(319)	(339)	(351)	(366)	(380)	(394)	(401)
Corporate tax rate		33%	33%	33%	33%	33%	33%	33%
NOPAT		647	687	714	742	771	801	815
D&A		131	132	139	144	150	155	157
(Net CAPEX)		(117)	(122)	(139)	(144)	(150)	(155)	(157)
(DWCR)		(139)	(144)	(144)	(144)	(144)	(144)	(144)
Free Cash Flow		**522**	**553**	**570**	**598**	**627**	**657**	**671**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**466**	**440**	**404**	**379**	**354**	**330**	**301**

Method

To obtain this DCF, we extended the business plan to a certain business horizon (2018) according to some assumptions. For instance :

1. The EBIT grows at a perpetuity rate after 2018.
2. The D& A and the CAPEX converge to be the opposite of each other in 2018.
3. The WCR is proportional to the sales.

After that, the cost of equity and the cost of debt are computed. An iterative loop is made to converge toward the Enterprise Value and the WACC.

Thanks to this method we obtain :

$$\begin{cases} EV = 5\ 877 \\ \\ WACC = 12.13\% \end{cases}$$

6.2 Continuous DCF

Collecting the data To perform the continuous DCF we have to collect certains values :

1. The EBIT dynamic : 906, 966, 1026, 1065, 1108, 1151, 1195, 1216.
2. The Net Debt : 210
3. The infinite change in WCR : $\Delta WCR_\infty = 144$
4. The cost of debt. Pre-tax : 3.18% Post-tax :2.13%
5. The cost of equity : 12.5%

Computing the growth rates

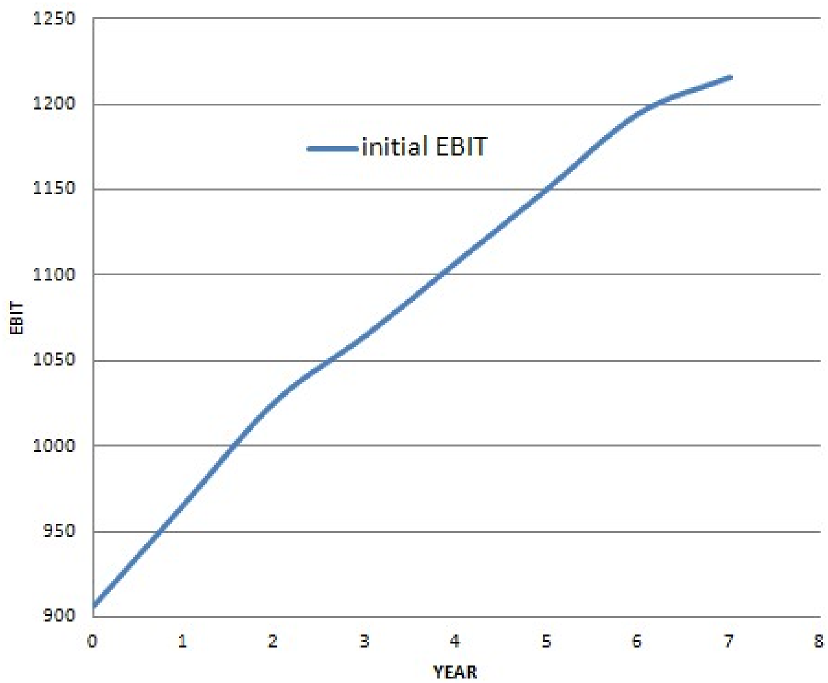
Then we compute the growth rates :

$$g_0 \sim \frac{\text{EBIT}_1}{\text{EBIT}_0} = \frac{966}{906} - 1 = 6.6\%$$

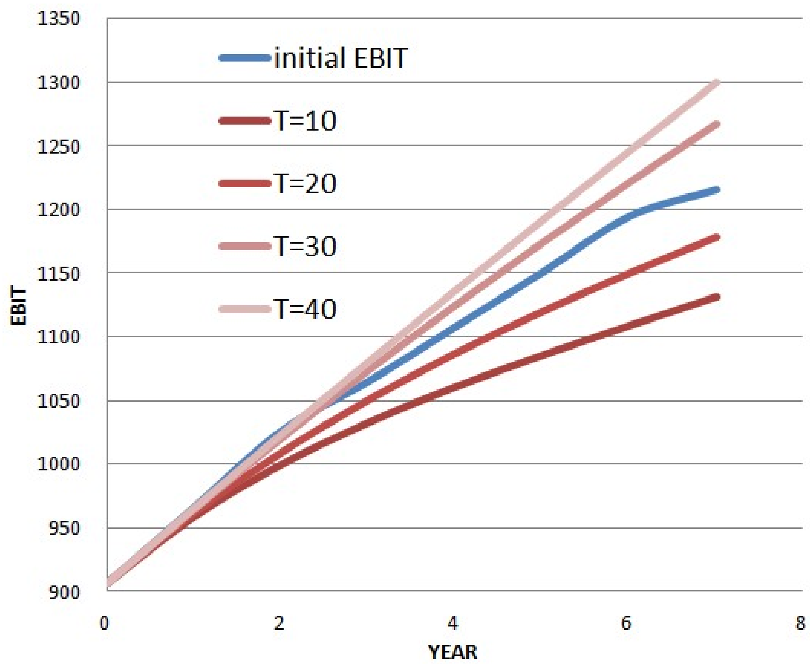
$$g_\infty = 2.5\% \text{ (Perpetuity growth rate)}$$

Calibration

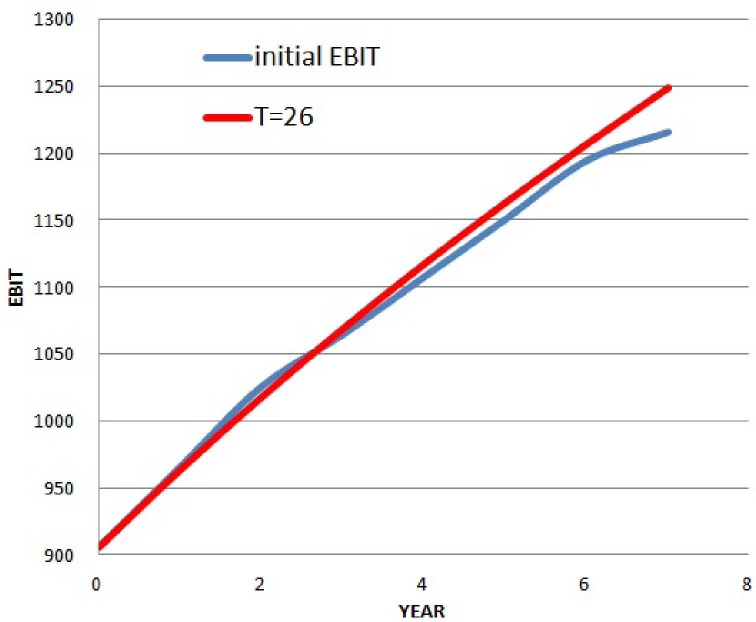
Here we carry out the calibration. The goal is to fit as well as possible the initial EBIT curve given by the business plan :



g_0 and g_∞ have been computed above. We try different values of T :

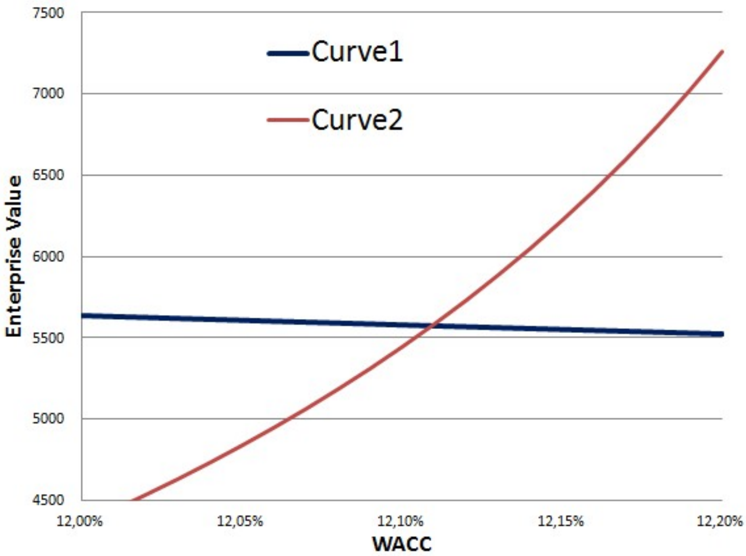


Finally, $T = 26$ is the best option, we obtain :



Solving the equations

Now that we have all the parameters we can draw the two curves of the system.



The result we obtain is :

$$\begin{cases} EV = 5\ 575 \\ \\ WACC = 12.11\% \end{cases}$$

6.3 Comparison

	Usual DCF	Continuous DCf
WACC	12,13%	12,11%
Enterprise Value	5877	5575

	Difference	
	Abs	Rel
WACC	0,02%	0,2%
Enterprise Value	302	5,1%

Like for Publicis, the results are good since the continuous DCF gives an Enterprise Value only 4.1% lower than the usual DCF. It is also included in the $\pm15\%$ error that we could tolerate for a DCF.

What is more, the WACC given by the continuous DCF is 0.5% higher than the discrete one. As we will see later, the relative error on the WACC is usualy very low.

7 Example : ALTEN

In this section we compare the results given by a usual DCF and the results given by the continuous DCF for Alten. In this case, we show that the model works for a negative net debt and that the initial growth rate g_0 can be calibrated as well to perform better results. Like above, we will focus on the accuracy of the Enterprise value and on the accuracy of the WACC.

7.1 Usual DCF

Risk free rate	3,50%
Beta (Datastream)	1,1
Unleveraged beta	1,15
Leveraged beta	1,14
Market risk premium	7,50%
Cost of equity (k)	**12,1%**
Beta of the debt	1,000
Cost of net debt	
Pretax (based on CAPM)	11,00%
Post tax	**7,03%**
WACC (K)	**12,32%**

Sum of discounted FCF	397
Terminal Value	415
Enterprise Value	**813**
Last net debt	(43)
Equity Value	**856**

Perpetuity growth rate	**2,0%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**107**	**110**	**120**	**126**	**131**	**135**	**139**	**142**
(Corporate tax on EBIT)		(40)	(43)	(45)	(47)	(49)	(50)	(51)
Corporate tax rate		36,10%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%
NOPAT		70	77	81	84	86	89	91
D&A		10	11	12	12	12	13	13
(Net CAPEX)		(10)	(10)	(11)	(11)	(12)	(12)	(13)
(DWCR)		6	8	8	7	6	5	4
Free Cash Flow		**76**	**86**	**90**	**92**	**92**	**95**	**95**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**68**	**68**	**63**	**58**	**52**	**47**	**42**

Method

To obtain this DCF, we extended the business plan to a certain business horizon (2018) according to some assumptions. For instance :

1. The EBIT grows at a perpetuity rate after 2018.
2. The D& A and the CAPEX converge to be the opposite of each other in 2018.
3. The WCR is proportional to the sales.

After that, the cost of equity and the cost of debt are computed. An iterative loop is made to converge toward the Enterprise Value and the WACC.

Thanks to this method we obtain :

$$\begin{cases} EV = 813 \\ WACC = 12.32\% \end{cases}$$

7.2 Continuous DCF

Collecting the data To perform the continuous DCF we have to collect certains values :

1. The EBIT dynamic : 107, 110, 120, 126, 131, 135, 139, 142.
2. The Net Debt : -43.
3. The infinite change in WCR : $\Delta WCR_{\infty} = -5$
4. The cost of debt. Pre-tax : 11.0% Post-tax :7.03%
5. The cost of equity : 12.1%

Computing the growth rates

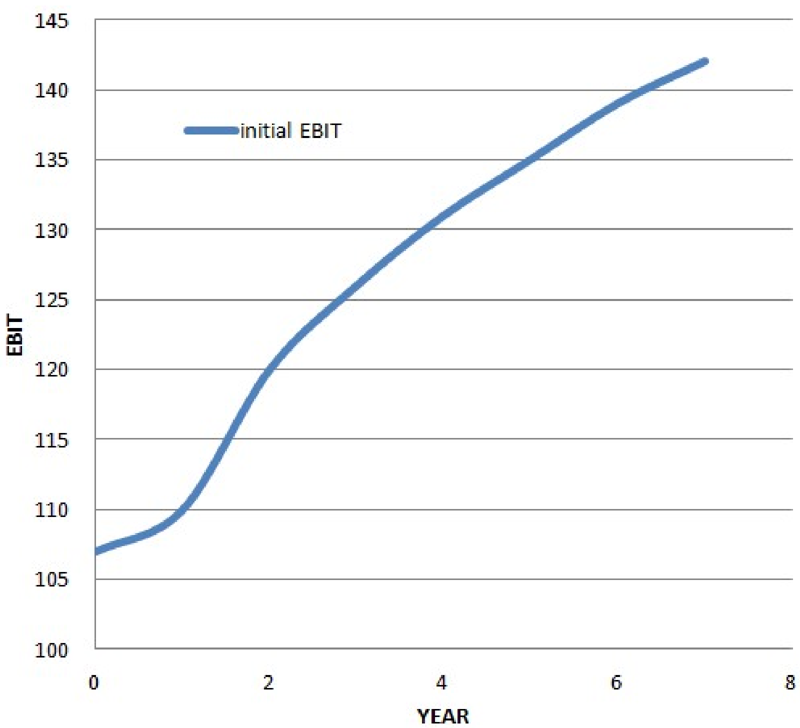
Then we compute the growth rates :

$$g_0 \sim \frac{\text{EBIT}_1}{\text{EBIT}_0} = \frac{110}{107} - 1 = 2.8\%$$

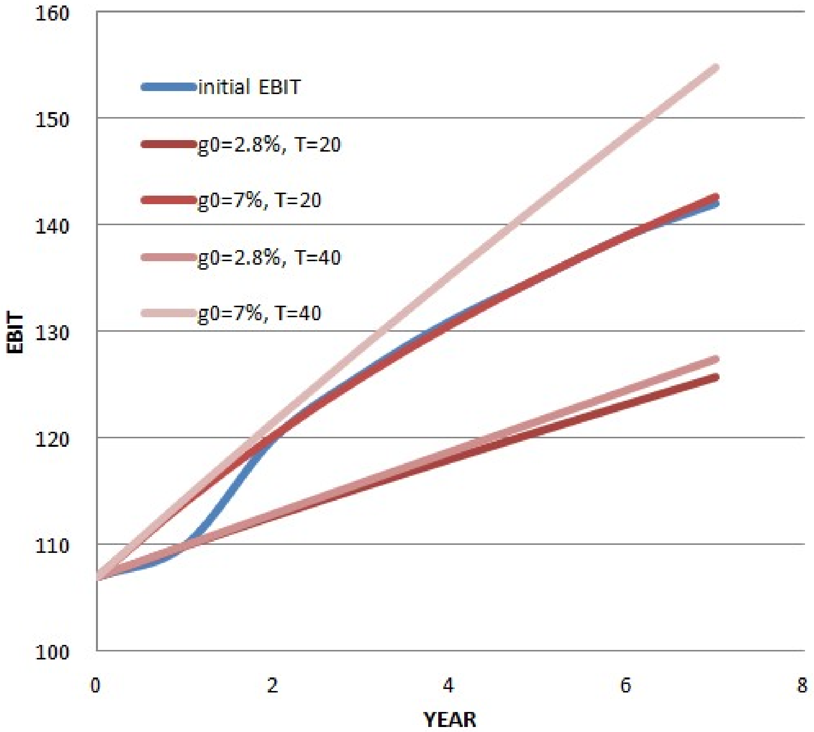
$$g_{\infty} = 2.0\% \text{ (Perpetuity growth rate)}$$

Calibration

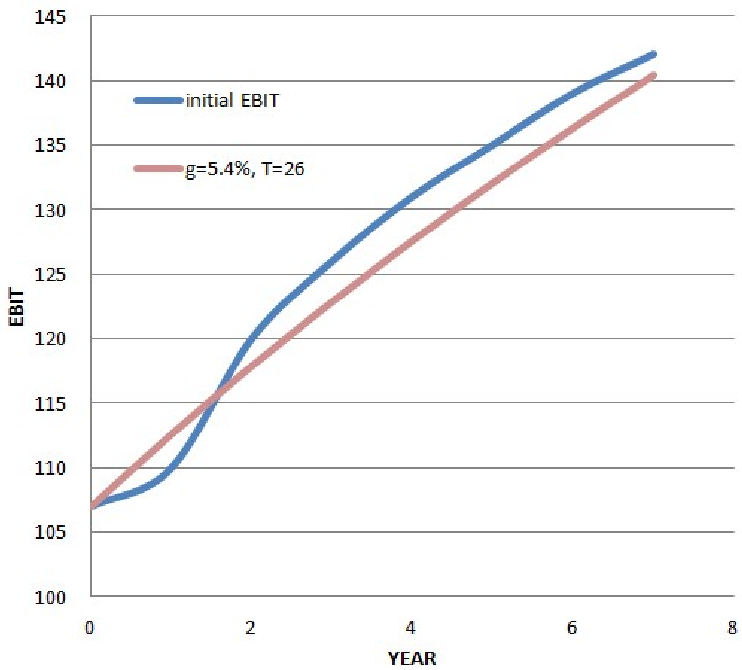
Here we carry out the calibration. The goal is to fit as well as possible the initial EBIT curve given by the business plan :



We see that the initial growth rate should be higher to fit the EBIT curve. We try different values of T and different values of g_0 :

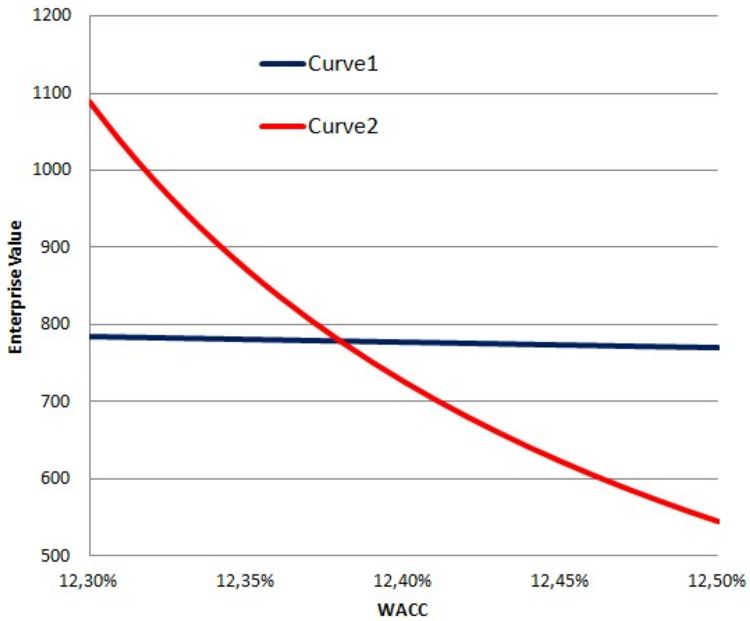


Finally, $T = 26$ and $g_0 = 5.4\%$ seems a good compromise betwenn fitting the curve at the beginning and at the end.



Solving the equations

Now that we have all the parameters we can draw the two curves of the system.



The result we obtain is :

$$\begin{cases} EV = 779 \\ \\ WACC = 12.38\% \end{cases}$$

7.3 Comparison

	Usual DCF	Continuous DCf
WACC	12,32%	12,38%
Enterprise Value	813	779

	Difference	
	Abs	Rel
WACC	0,06%	0,5%
Enterprise Value	34	4,1%

The results are pretty good since the continuous DCF gives an Enterprise Value only 5.1% higher than the usual DCF. It is included in the ±15% error we could tolerate for a DCF.

What is more, the WACC given by the continuous DCF is 0.2% higher than the discrete one. We are probably quite lucky to obtain such a good precision. However, as we will see later, the relative error on the WACC is usualy less important than on the Enterprise Value.

8 Example : ARKEMA

In this section we compare the results given by a usual DCF and the results given by the continuous DCF for Arkema. In this case, we show that the model works when the part of the Net Debt in the Enterprise Value is quite significant and when the cost of debt is really lower than the cost of equity. Like above, we will focus on the accuracy of the Enterprise value and on the accuracy of the WACC.

8.1 Usual DCF

Risk free rate	3,50%
Beta (Datastream)	-
Unleveraged beta	1,1
Leveraged beta	1,2
Market risk premium	7,50%
Cost of equity (k)	**12,5%**
Beta of the debt	-
Cost of net debt	
Pretax (based on CAPM)	3,18%
Post tax	**2,10%**
WACC (K)	11,16%

Sum of discounted FCF	2 066
Terminal Value	3 033
Enterprise Value	**5 099**
Last net debt	658
Equity Value	**4 441**

Perpetuity growth rate	**2,5%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**576**	**630**	**688**	**722**	**757**	**792**	**829**	**867**
(Corporate tax on EBIT)		(214)	(234)	(245)	(257)	(269)	(282)	(295)
Corporate tax rate		34%	34%	34%	34%	34%	34%	34%
NOPAT		415	454	476	499	523	547	572
D&A		289	291	303	315	328	342	356
(Net CAPEX)		(336)	(332)	(341)	(349)	(358)	(367)	(376)
(DWCR)		27	(45)	(14)	(14)	(15)	(15)	(15)
Free Cash Flow		**396**	**368**	**425**	**451**	**479**	**508**	**537**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**356**	**298**	**309**	**296**	**282**	**269**	**256**

Method

To obtain this DCF, we extended the business plan to a certain business horizon (2018) according to some assumptions. For instance :

1. The EBIT grows at a perpetuity rate after 2018.
2. The D& A and the CAPEX converge to be the opposite of each other in 2018.
3. The WCR is proportional to the sales.

After that, the cost of equity and the cost of debt are computed. An iterative loop is made to converge toward the Enterprise Value and the WACC.

Thanks to this method we obtain :

$$\begin{cases} EV = 5\ 099 \\ \\ WACC = 11.16\% \end{cases}$$

8.2 Continuous DCF

Collecting the data To perform the continuous DCF we have to collect certains values :

1. The EBIT dynamic : 576, 630, 688, 722, 757, 792, 829, 867.
2. The Net Debt : 658.
3. The infinite change in WCR : $\Delta WCR_{\infty} = 15$
4. The cost of debt. Pre-tax : 3.18% Post-tax :2.10%
5. The cost of equity : 12.5%
6. The tax rate : 34.0%

Computing the growth rates

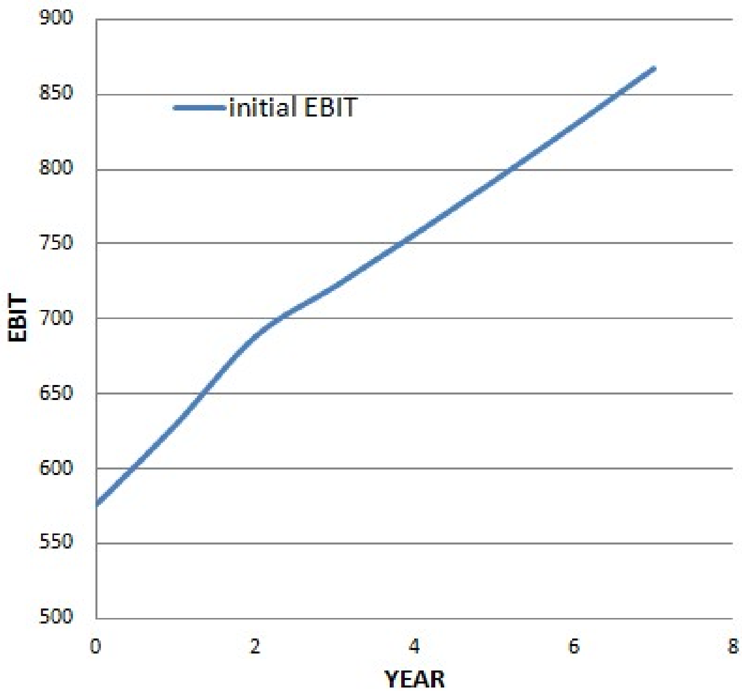
Then we compute the growth rates :

$$g_0 \sim \frac{\text{EBIT}_1}{\text{EBIT}_0} = \frac{629.5}{576} - 1 = 6.2\%$$

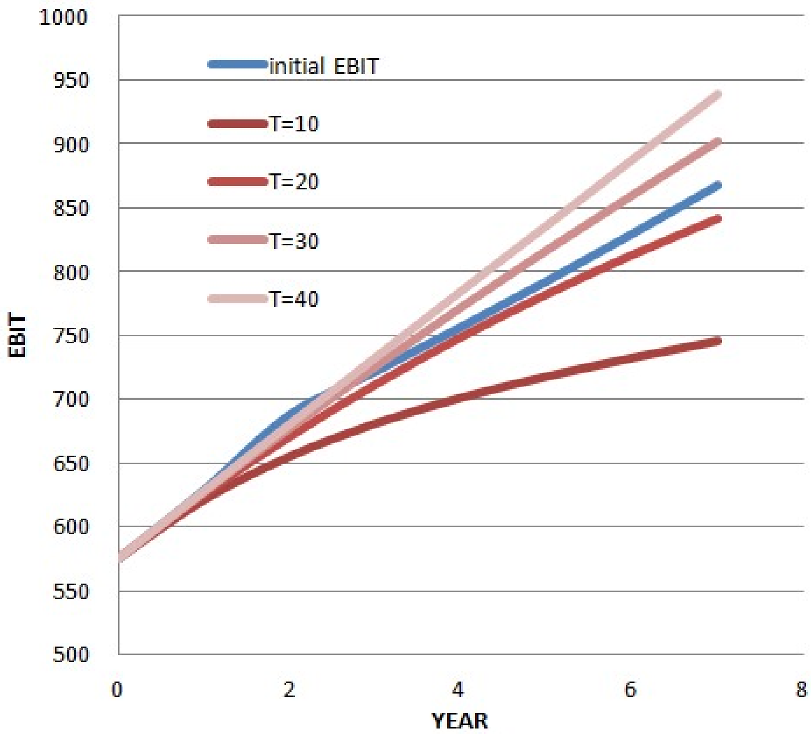
$$g_{\infty} = 2.5\% \text{ (Perpetuity growth rate)}$$

Calibration

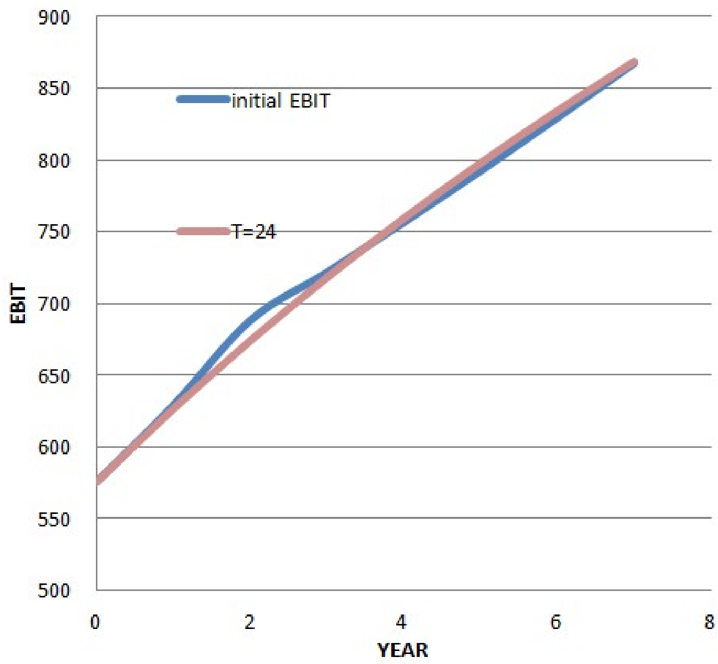
Here we carry out the calibration. The goal is to fit as well as possible the initial EBIT curve given by the business plan :



We try different values of T :

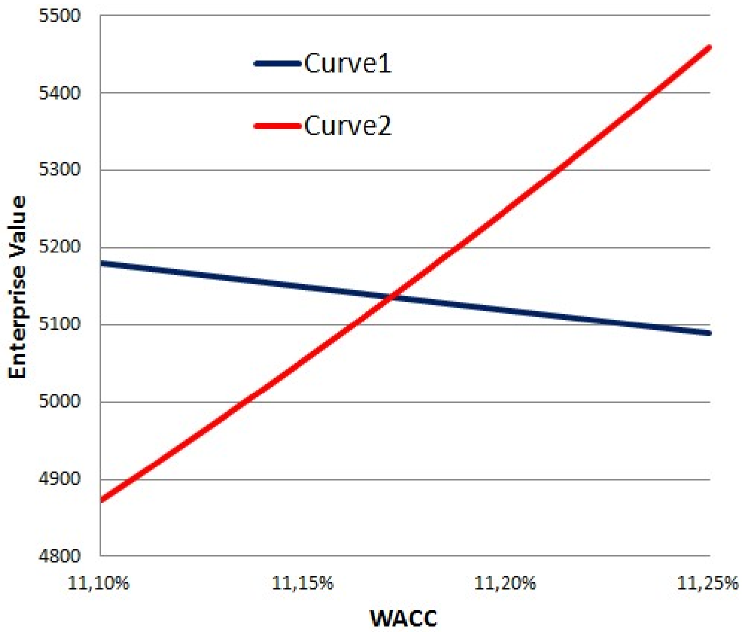


Finally, $T = 26$ is the best option, we obtain :



Solving the equations

Now that we have all the parameters we can draw the two curves of the system.



The result we obtain is :

$$\begin{cases} EV = 5134 \\ \\ WACC = 11.17\% \end{cases}$$

8.3 Comparison

	Usual DCF	Continuous DCf
WACC	11,16%	11,17%
Enterprise Value	5099	5134

	Difference	
	Abs	Rel
WACC	0,01%	0,1%
Enterprise Value	35	0,7%

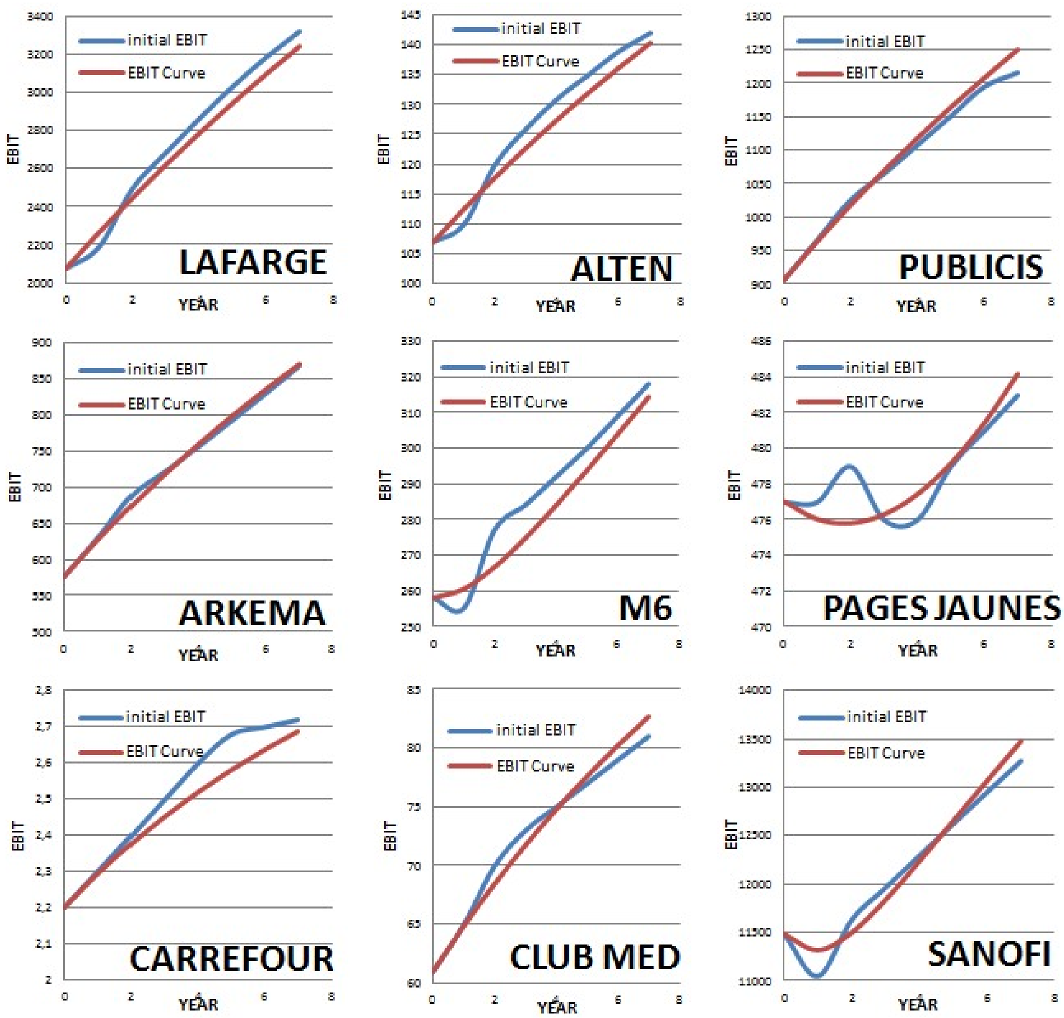
This time, the Enterprise Value and the WACC are both under the 1% level of error. The Enterprise Value is 0.7% higher with the continuous DCF. The WACC is 0.1% higher with the continuous DCF. We will see later that this may be correlated to the fact that the part of the Net Debt is quite important in the Enterprise Value. However, obtaining such a good result is probably a little bit due to luck.

9 Testing the model validity

To evaluate the validity of the model, we have carried 10 continuous DCF and we have compared them to the result given by a classical DCF. The companies have been chosen randomly. They are of different sizes and of different indebtness levels. The size of the company should not be a limit since it is only a scale parameter : working in millions or in billions does not change the method. We will define the limits of this model in the last part.

9.1 Calibration

The calibration is the corner stone of the continuous DCF. It designs the EBIT which is probably the most important term in the computation of the Free Cash Flows. See below the calibration curves from 9 of the 10 models.



As we can see, in every case the model curve can almost match the initial EBIT curve. After 5 years, the growth is quite constant. The convergence toward the perpetuity growth rate tends to crush the volatility of the groth. However, in the first couple of years, the movements are more stochastic and harder to fit. It is especially the case of M6, Sanofi or Pages Jaunes. We manage those erractic growths by taking their average.

9.2 Results of the 10 tests

The first table below contains the results of the two DCF methods for the Enterprise Value and the WACC. The second table present the absolute and relative differences of those results. The relative difference is computed as follows : Relative Diff$_{EV}$ = $\frac{EV_{\text{continuous}}-EV_{\text{usual}}}{EV_{\text{usual}}}$. Same for the WACC.

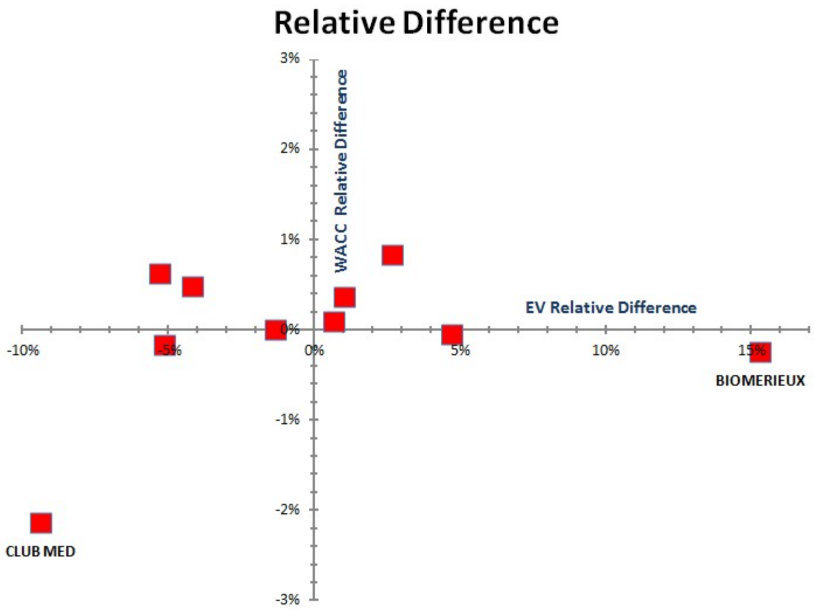
RESULTS

		Enterprise Value		WACC	
		Usual DCf	Continuous DCf	Usual DCf	Continuous DCf
1	LAFARGE	29471	29780	8,06%	8,09%
2	ALTEN	813	779	12,32%	12,38%
3	PUBLICIS	5877	5575	12,13%	12,11%
4	ARKEMA	5099	5134	11,16%	11,17%
5	M6	1762	1669	12,80%	12,88%
6	PAGES JAUNES	5493	5640	5,98%	6,03%
7	CARREFOUR (bn)	22,6	22,4	8,55%	8,55%
8	CLUB MED	631	571,8	10,31%	10,09%
9	SANOFI	101244	106035	9,40%	9,40%
10	BIOMERIEUX	2324	2680	9,58%	9,56%

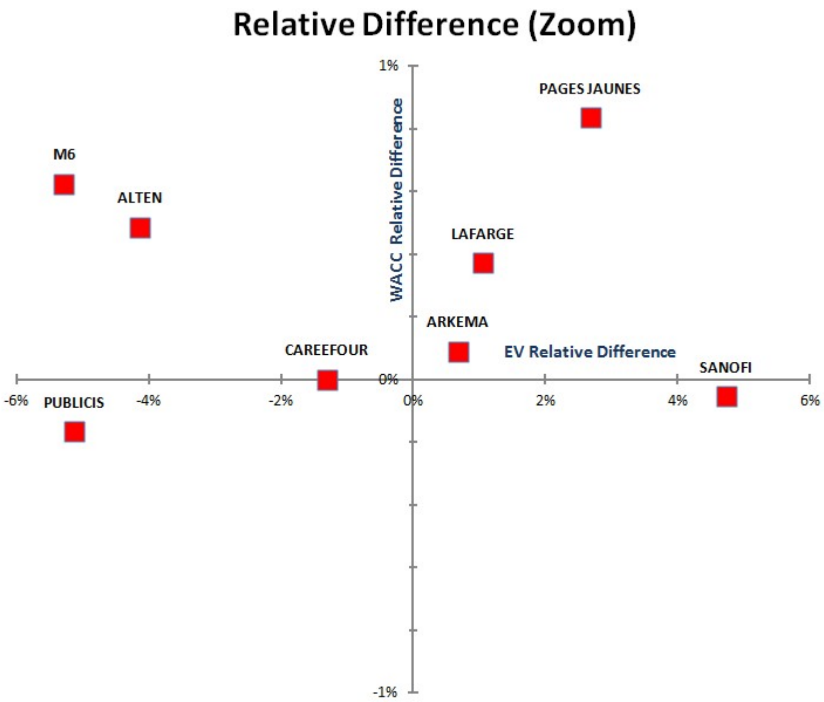
DIFFERENCES

		Enterprise Value diff		WACC diff	
		Absolute	Relative	Absolute	Relative
1	LAFARGE	309	1,0%	0,03%	0,37%
2	ALTEN	-34	-4,1%	0,06%	0,49%
3	PUBLICIS	-302	-5,1%	-0,02%	-0,16%
4	ARKEMA	35	0,7%	0,01%	0,09%
5	M6	-93	-5,3%	0,08%	0,62%
6	PAGES JAUNES	147	2,7%	0,05%	0,84%
7	CARREFOUR (bn)	-0,3	-1,3%	0,00%	0,00%
8	CLUB MED	-59	-9,4%	-0,22%	-2,13%
9	SANOFI	4791	4,7%	0,00%	-0,05%
10	BIOMERIEUX	356	15,3%	-0,02%	-0,25%
	AVERAGE	-	-0,08%	-	-0,02%

We have drawn below the graphic with the EV relative difference in abscissa and the WACC relative difference in ordinate.



Here is a zoom on the results when we remove the two extreme points : Club Med and Biomérieux.



Analysis of the results

The first conclusion we may draw from those results is that the average of the reltive difeerences are excellent. The average relative difference for the Enterprise Value is -0.08%. On the other side, the average relative difference for the WACC is -0.02%. We will later focus more deeply on the average relative difference with the Student test and estimate a 95%-confidence interval for the average relative difference.

Besides, the relative difference is acceptable for each company. For the Enterprise Value, the extreme cases are Club Med (-9.3%) and Biomerieux ($+15.4\%$). The case of Biomerieux is quite special since in the usual DCF, the Capex and D& A do not compensate each other on the long run. This is a debatable choice. Otherwise for the other companies the Enterprise Values relative differences are under the 5% margin of error. The WACC is always well computed.

We notice that the error on the Enterprise Value is relatively more significant than on the WACC. Finding an explanation for this statement is quite difficult. This may be due to the slope of each curve or this can also be due to the fact that we do not modify the second equation of the system.

The results for the EV seem equally distributed on the positive and the negative side. On the other hand, the results for the WACC seem to be more often positive. However, sample used is quite small and in any case the relative difference for the WACC is too small to really care about its sign.

9.3 Student test

Theory

Each test gives a relative difference for the Enterprise Value and a relative difference for the WACC. Let's assume that those results are two normal random variables. This assumption seems reasonable : the relative difference can hardly be foreseen and the companies have been chosen randomly. On top of that, if we peek to the distribution obtained with the 10 tests, we guess a certain concentration around a given mean and a certain variance.

Assuming that the two relative differences are normally distributed, the Student Test allows us to obtain the distribution of the mean for each random variable.

We recall here the theorem of the Student Distribution.

Theorem : Let x_1, ..., x_n be the numbers observed in a sample from a normal distribution $x \sim N(\mu, \sigma^2)$. The sample mean and sample variance are respectively :

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$S = \tfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

Then $t = \frac{\overline{x}-\mu_0}{\sqrt{S/n}}$ follows a Student Law with $n-1$ degrees of freedom.

We deduce that $\mu_0 = \overline{x} + t\sqrt{S/n}$ follows a Student Law with $n-1$ degrees of freedom, centered in $\overline{x}$ and dilated by a factor $\sqrt{S/n}$. This theoretical distribution will be drawn below.

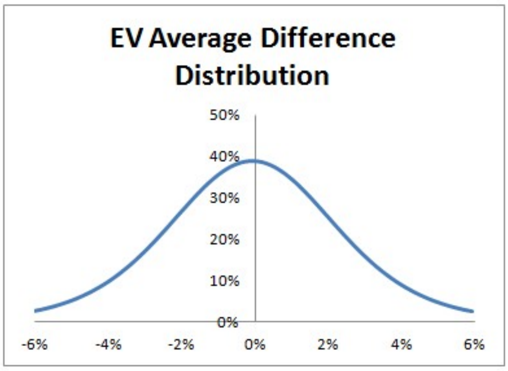
The α-confidence interval for μ_0 is given by :

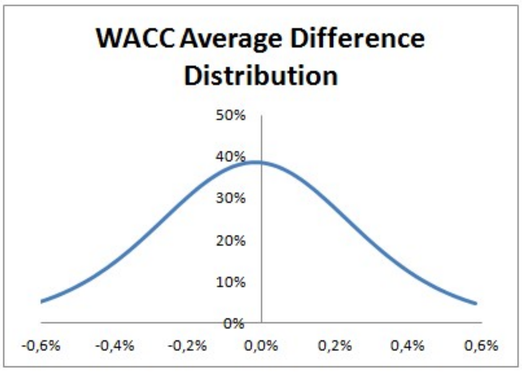
$$\left[\overline{x} - t^{n-1}_{(1-\alpha)/2}\sqrt{\frac{S}{n}}, \overline{x} + t^{n-1}_{(1-\alpha)/2}\sqrt{\frac{S}{n}}\right]$$

Where t^k_γ is the γ-quantile of the Student Law with k degrees of freedom.

95% confidence interval for the mean

According to the previous theorem, we can draw the distribution probability of the mean of the relative difference $\mu_0 = \overline{x} + t\sqrt{S/n}$ ($n = 10$, $\overline{x}$ is the empirical mean, and t is a 9-Student Law). We do it for the average of the relative difference of the Enterprise Value and we do it for the average of the relative difference of the WACC. The distributions presented below are two Student Laws with $n - 1 = 9$ degrees of freedom calibrated with the results of the 10 tests. As the reader may know, a Student Law has two parameters : the mean and the number of degrees of freedom. Unlike the normal law, there is no specific parameter for the variance. It explains why those two distributions have the same shape even if they are at two different scales.





We compute below the 95% confidence interval of the mean of the relative difference for the Enterprise Value and for the WACC. The 95% bilateral quantile of the Student Law is equal to 2.26.

EV	AVERAGE	-0,08%
	VARIANCE	0,0047
	OBSERVATIONS	10

	95% Confidence Interval	
EV	min	-5,00%
	max	4,84%

WACC	AVERAGE	-0,02%
	VARIANCE	0,0001
	OBSERVATIONS	10

	95% Confidence Interval	
WACC	min	-0,61%
	max	0,57%

For the Enterprise Value we see that the continuous DCF model has a 95% probability of having an average error between -5.0% and 4.84%. On the other hand, the continuous DCF model has a 95% probability of having an average error between -0.61% and 0.57%. This excellent intervals confirm what we expected at the sight of the results. This model does not insert a significant bias in the computation of the Enterprise Value nor in the computation of the WACC.

Of course we balance our opinion and we recall that we are under the assumption that the relative error is normally distributed. We recall as well that this result does not mean that each test will be between -5% and 4.84% for the Enterprise Value with a 95% probability. It means that the average of this error, on one millions tests for instance, has a 95% probability to be in this interval. On the whole, we expected to get a good accuracy but we did not expect so good results. The next part is devoted to finding the limits and the framework within which this model can be used.

The Aspin-Welch test

The aim of this section which provides the same results than above is to present the conclusion in a way easy to understand for people used to the statistical analysis.

We recall that we have two random variables : the relative difference of the Enterprise Value and the relative difference of the WACC. It would be useless to test the absolute values of the two DCFs since each company leads to its proper valuation. That way we compare these two relative differences to a random variable equal to zero (that is to say a Dirac Variable centered in Zero).

Our goal is to estimate whether or not the continuous DCF introduce a bias in the computation of the Enterprise Value of in the computation of the WACC. To that end, we use the Aspin-Welch test, it is a test of equality for two means when the variances are different. We can say that the variances are difference since the Variable 1 is a Dirac and the Variable 2 is not. Due to the fact that the variance of a Dirac is zero ($S_y = 0$), we can say that t follows a Student Law with $n_x - 1 = 9$ degrees of freedom :

$$t = \frac{\overline{x} - \mu_0}{\sqrt{\frac{S_x}{n_x} + \frac{S_y}{n_y}}} = \frac{\overline{x} - \mu_0}{\sqrt{S_x/n_x}}$$

$$\text{Degrees of Freedom} = \mathbb{E}\Big(\frac{(\frac{S_x}{n_x} + \frac{S_y}{n_y})^2}{(\frac{S_x}{n_x})^2/(n_x - 1) + (\frac{S_y}{n_y})^2/(n_y - 1)}\Big) = n_x - 1$$

Here are the results obtained for the two tests :

Enterprise Value
Test for equality of two means: observations with different variances

	Variable 1	*Variable 2*
Average	0	-0,0008
Variance	0	0,0047
Observations	10	10
Degrees of Liberty	9	
Statistic t	**0,04**	
P(T<=t) bilatéral	0,97	
Critical Value of t (bilatéral)	**2,26**	

WACC
Test for equality of two means: observations with different variances

	Variable 1	*Variable 2*
Average	0	-0,00019
Variance	0	6,8E-05
Observations	10	10
Degrees of Liberty	9	
Statistic t	**0,07**	
P(T<=t) bilatéral	0,94	
Critical Value of t (bilatéral)	**2,26**	

Since $0.04 < 2.26$ and $0.07 < 2.26$ we can say that on the 10 tests basis and with a 95% probability there is no bias for the Enterprise Value and the WACC, i.e. the means are equal to zero.

10 Limits of the continuous DCF

10.1 Regressions - source of error

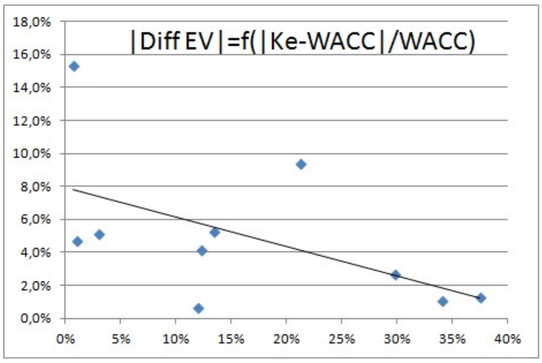
Distance WACC - Cost of Equity (k_e)

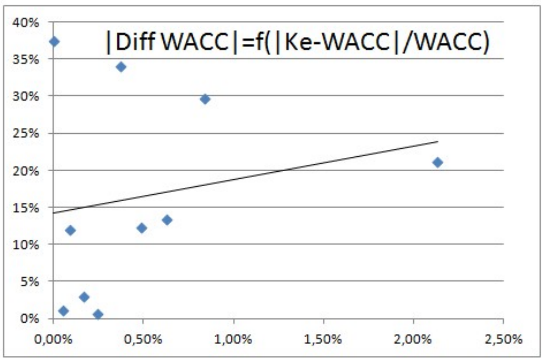
The system which defines the Enterprise Value and the WACC is the following :

$$\begin{cases} EV = (1-\tau)EBIT_0(\frac{1}{k-g_\infty} + \frac{1}{k+\lambda-g_0} - \frac{1}{k+\lambda-g_\infty}) - \frac{\Delta WCR_\infty}{k-g_\infty} \\ EV = ND\frac{k_d(1-\tau)-k_e}{WACC-k_e} \end{cases}$$

The second equation explodes when the WACC tends to the Cost of Equity k_e. This should imply that the error is more significant when the WACC is relatively close to the Cost of Equity. To check this intuition, with have drawn the regression of the Relative Difference as a function of the relative distance between the WACC and the Cost of Equity. We did such a computation for the Enteprise Value and the WACC as well.

		Relative Difference		|Ke-WACC|	|Ke-WACC| /WACC
		EV	WACC		
1	LAFARGE	1,0%	0,37%	2,74%	34%
2	ALTEN	4,1%	0,49%	0,27%	12%
3	PUBLICIS	5,1%	0,16%	0,37%	3%
4	ARKEMA	0,7%	0,09%	1,34%	12%
5	M6	5,3%	0,62%	1,71%	13%
6	PAGES JAUNES	2,7%	0,84%	1,78%	30%
7	CARREFOUR (bn)	1,3%	0,00%	3,20%	37%
8	CLUB MED	9,4%	2,13%	2,19%	21%
9	SANOFI	4,7%	0,05%	0,10%	1%
10	BIOMERIEUX	15,3%	0,25%	0,06%	1%
	AVERAGE	5,0%	0,50%	-	16,49%





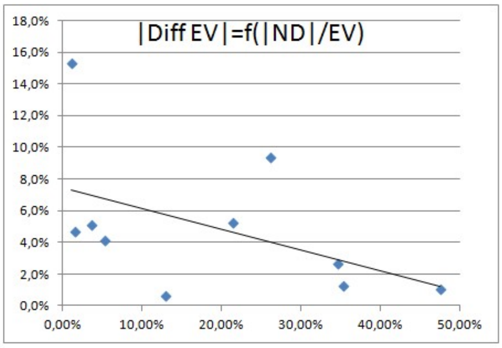
For the Enterprise Value it seems quite clear that the relative distance between the WACC and the Cost of Equity has an impact on the accuracy of our model. The more this relative distance is important, the more accurate is the result. When this distance is around 30% or 40% the absolute error seems to be under the 3%, which is very good. Of course, we balance this conclusion by the small size of the sample.

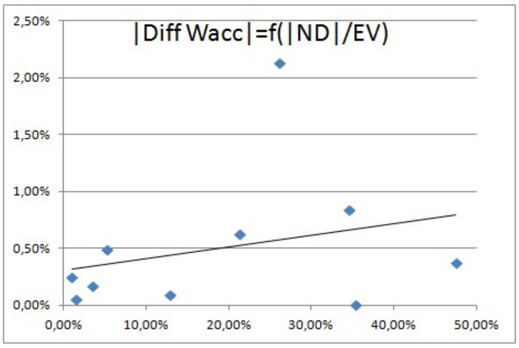
On the other hand, this conclusion does not seem valid for the WACC. The relative error on the WACC does not really depend on the relative distance between the WACC and the Cost of Equity. This analysis is less important than the previous one since the relative error is very small.
To explain mathematically this result, we can say that the slope of the curve given by the second equation is high when the WACC is close to the cost of equity. An error on a quite vertical curve has an impact on the ordinate and not on the abscissa. That is why the relative difference on the Enterprise Value is more sensible to this situation than the WACC.

Indebtedness

Having a WACC relatively close to the Cost of Equity means that the Net Debt is low in absolute value. We have drawn below the regression showing the relative difference as a function of the indebtedness (i.e. $\frac{\text{Net Debt}}{\text{Enterprise Value}}$).

		Relative Difference		|ND|/EV
		EV	WACC	
1	LAFARGE	1,0%	0,37%	47,48%
2	ALTEN	4,1%	0,49%	5,29%
3	PUBLICIS	5,1%	0,16%	3,57%
4	ARKEMA	0,7%	0,09%	12,90%
5	M6	5,3%	0,62%	21,39%
6	PAGES JAUNES	2,7%	0,84%	34,59%
7	CARREFOUR (bn)	1,3%	0,00%	35,33%
8	CLUB MED	9,4%	2,13%	26,15%
9	SANOFI	4,7%	0,05%	1,56%
10	BIOMERIEUX	15,3%	0,25%	1,08%
	AVERAGE	5,0%	0,50%	18,93%





The results of those two regressions are quite the same than for the relative distance WACC - Cost of Equity. Our model provides better results when the indebtedness is not insignificant.

10.2 Limits of the assumptions

1. The EBIT grows at a constant rate g0 at the begining and at a rate g1 on the long run

This assumption is the corner stone of our model. The 10 examples taken randomly seems to validate the idea that the EBIT dynamic can be approximated by 3 parameters : the initial growth rate, the perpetuity growth rate, and the period of transition between the two. The EBIT is more volatile at the beginning but after the erratic movements are crushed by the convergence toward the perpetuity growth rate. This statement has to be tested for smaller companies like startups which encounter a more hazardous EBIT evolution.

2. The D& A and the CAPEX compensate each other

This assumption helps us to simplify the first equation of the system. This statement is often valid around the business horizon and the D& A and the CAPEX compensate each other in the terminal value. However, the difference could be quite significant in some extreme cases. A user should keep in mind this assumption.

3. The Change in WCR grows at the rate g_∞

The change in WCR is not really significant compared to the other numbers, especially the EBIT or the taxes. However, to obtain a better precision, we can model it with a constant growth rate. This assumption seems to be precise enough to end up with good final results.

4. The WACC is constant over time

This assumption is widely used, even for the usual DCF method. Of course, a company is not supposed to finance itself at the same rate during all its future life. Regarding the Cost of Equity, the market risk premium and the risk free rate can change as time goes by. The Cost of Debt is also a function of the indebtedness of the company, a number obviously not constant. However, this approximation is quite always used and the WACC can ben interpreted as an average over time. In some very rare cases, for a startup for instance, it can be useful to use a WACC depending on the time.

11 Conclusion

On the whole, we can draw modestly two conclusions from this work.

Firstly, to compute a DCF wisely it seems vital to understand the precise role of the FCF and the WACC. More precisely, the WACC equation has a severe impact on the result. Basically, computing the WACC must be made with the same seriousness than for computing the Free Cash Flows.

Secondly, under some realistic assumptions, it is possible to obtain good results for the Enterprise Value and the WACC more quickly than usual. The continuous DCF methods uses less parameters than the usual DCF method without losing too much precision. What is more, rather than relying a quite obscure loop, this model works in a way by which the user really understands the role played by each equation. In fact, the solution is the intersection of two curves defined by the system governing the DCF.

However, we must acknowledge that writing all the parameters for a usual DCF seems more reassuring for the analyst and that more tests have to be made in order to really validate the accuracy of the continuous DCF method.

12 Appendix

Find in the following pages the details of the 10 (usual and continuous) DCF computed for the test of validity. The companies are :

1. Lafarge
2. Alten
3. Publicis
4. Arkema
5. M6
6. Pages Jaunes
7. Carrefour
8. Club Med
9. Sanofi
10. Biomérieux

LAFARGE

Risk free rate	4,50%
Beta (Datastream)	1,35
Unleveraged beta	0,70
Leveraged beta	0,84
Market risk premium	7,50%
Cost of equity (k)	**10,8%**
Beta of the debt	0,450
Cost of net debt	
Pretax (based on CAPM)	7,88%
Post tax	**5,03%**
WACC (K)	**8,06%**

Sum of discounted FCF	8 808
Terminal Value	20 663
Enterprise Value	**29471**
Last net debt	13 993
Equity Value	**15 478**

Perpetuity growth rate	**2,0%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**2 071**	**2 185**	**2 494**	**2 682**	**2 862**	**3 031**	**3 185**	**3 321**
(Corporate tax on EBIT)		(789)	(900)	(968)	(1 033)	(1 094)	(1 150)	(1 199)
Corporate tax rate		36,10%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%
NOPAT		1 396	1 594	1 714	1 829	1 937	2 035	2 122
D&A		1 108	1 112	1 210	1 306	1 398	1 484	1 563
(Net CAPEX)		(1 065)	(1 268)	(1 327)	(1 386)	(1 445)	(1 504)	(1 563)
(DWCR)		(6)	(24)	(22)	(20)	(17)	(14)	(10)
Free Cash Flow		**1 433**	**1 414**	**1 575**	**1 729**	**1 873**	**2 001**	**2 112**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**1 326**	**1 211**	**1 248**	**1 268**	**1 271**	**1 257**	**1 228**

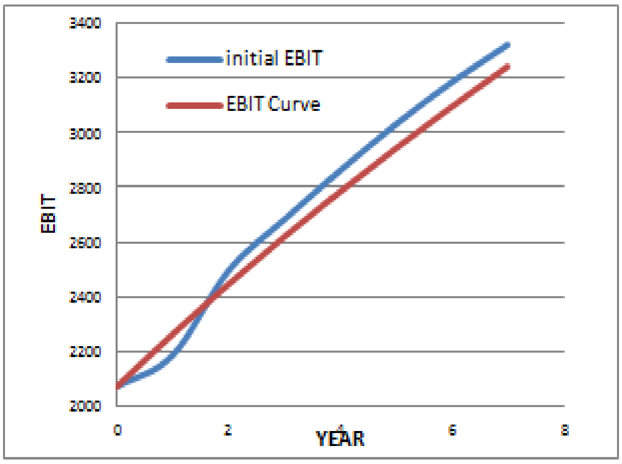
LAFARGE

Parameters

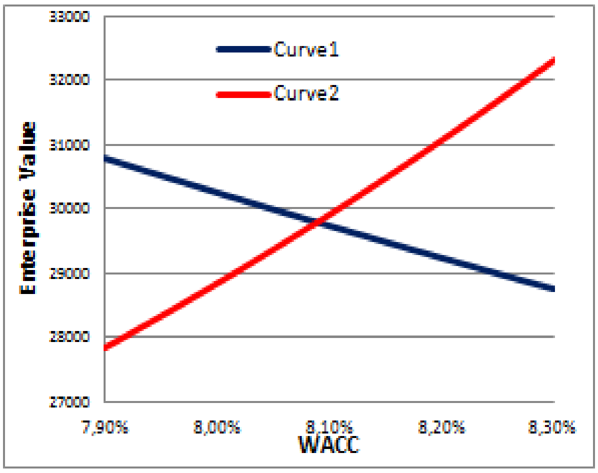
Curve 1	
initial EBIT (EBIT_0)	2071
initial growth (g_0)	9,50%
Perpetuity growth (g_inf)	2%
Transition Period (T)	30
Tax Rate (tau)	36%
DWCR	10

Curve 2	
Net Debt	13993
Cost of Equity	10,80%
Cost of Debt (post t	5,03%

Calibration



Valuation



Results

	Usual DCF	Continuous DCf
WACC	8,06%	8,09%
Enterprise Value	29471	29780

	Difference	
	Abs	Rel
WACC	0,03%	0,4%
Enterprise Valu	309	1,0%

Comment:

Excellent relative difference. Good Calibration. The indebtedness of this company is quite important.

ALTEN

Risk free rate	3,50%
Beta (Datastream)	1,1
Unleveraged beta	1,15
Leveraged beta	1,14
Market risk premium	7,50%
Cost of equity (k)	**12,1%**
Beta of the debt	1,000
Cost of net debt	
Pretax (based on CAPM)	11,00%
Post tax	**7,03%**
WACC (K)	**12,32%**

Sum of discounted FCF	397
Terminal Value	415
Enterprise Value	**813**
Last net debt	(43)
Equity Value	**856**

Perpetuity growth rate	**2,0%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**107**	**110**	**120**	**126**	**131**	**135**	**139**	**142**
(Corporate tax on EBIT)		(40)	(43)	(45)	(47)	(49)	(50)	(51)
Corporate tax rate		36,10%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%
NOPAT		70	77	81	84	86	89	91
D&A		10	11	12	12	12	13	13
(Net CAPEX)		(10)	(10)	(11)	(11)	(12)	(12)	(13)
(DWCR)		6	8	8	7	6	5	4
Free Cash Flow		**76**	**86**	**90**	**92**	**92**	**95**	**95**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**68**	**68**	**63**	**58**	**52**	**47**	**42**

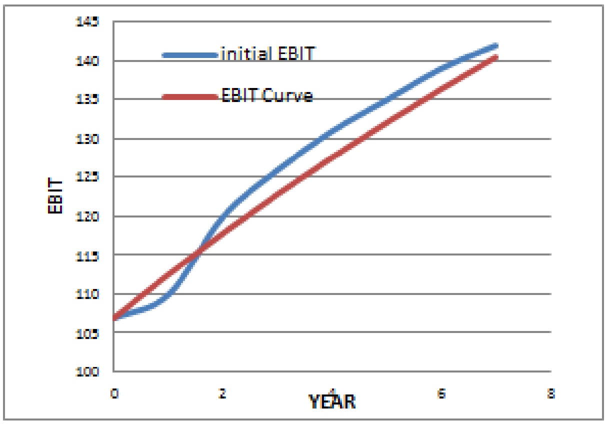
ALTEN

Parameters

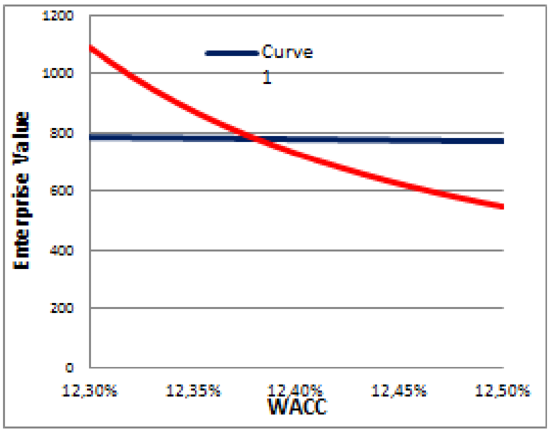
Curve 1	
initial EBIT (EBIT_0)	107
initial growth (g_0)	5,4%
Perpetuity growth (g_inf)	2%
Transition Period (T)	30
Tax Rate (tau)	36%
DWCR	-5

Curve 2	
Net Debt	-43
Cost of Equity	12,10%
Cost of Debt (post t	7,03%

Calibration



Valuation



Results

	Usual DCF	Continuous DCf
WACC	12,32%	12,38%
Enterprise Value	813	779

	Difference Abs	Difference Rel
WACC	0,06%	0,5%
Enterprise Valu	34	4,1%

Comment:

Excellent relative difference for the WACC. The one for the Enterprise value is correct. Good calibration. This company has a negative net debt.

PUBLICIS

Risk free rate	3,50%
Beta (Datastream)	-
Unleveraged beta	1,1
Leveraged beta	1,2
Market risk premium	7,50%
Cost of equity (k)	**12,5%**
Beta of the debt	-
Cost of net debt	
Pretax (based on CAPM)	3,18%
Post tax	**2,13%**
WACC (K)	**12,13%**

Sum of discounted FCF	2 674
Terminal Value	3 203
Enterprise Value	**5 877**
Last net debt	210
Equity Value	**5 667**

Perpetuity growth rate	**2,5%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**906**	**966**	**1 026**	**1 065**	**1 108**	**1 151**	**1 195**	**1 216**
(Corporate tax on EBIT)		(319)	(339)	(351)	(366)	(380)	(394)	(401)
Corporate tax rate		33%	33%	33%	33%	33%	33%	33%
NOPAT		647	687	714	742	771	801	815
D&A		131	132	139	144	150	155	157
(Net CAPEX)		(117)	(122)	(139)	(144)	(150)	(155)	(157)
(DWCR)		(139)	(144)	(144)	(144)	(144)	(144)	(144)
Free Cash Flow		**522**	**553**	**570**	**598**	**627**	**657**	**671**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**466**	**440**	**404**	**379**	**354**	**330**	**301**

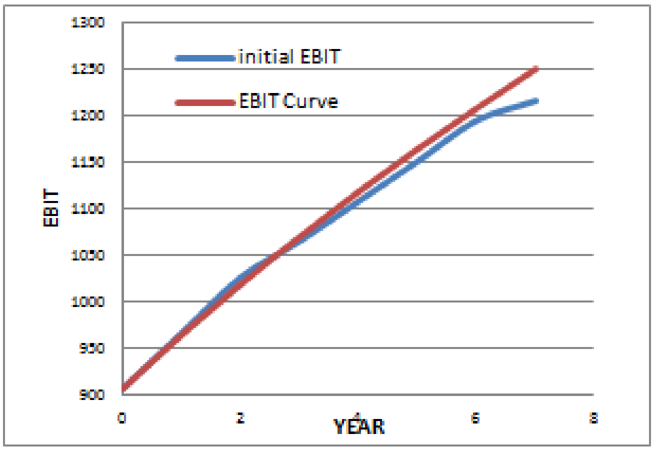
PUBLICIS

Parameters

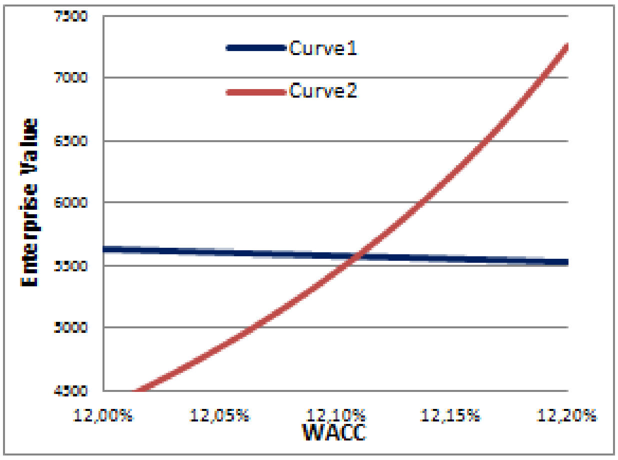
Curve 1	
initial EBIT (EBIT_0)	906
initial growth (g_0)	6,6%
Perpetuity growth (g_inf)	2,5%
Transition Period (T)	26
Tax Rate (tau)	33,0%
DWCR	144

Curve 2	
Net Debt	210
Cost of Equity	12,50%
Cost of Debt (post tax)	2,13%

Calibration



Valuation



Results

	Usual DCF	Continuous DCF
WACC	12,13%	12,11%
Enterprise Value	5877	5575

	Difference	
	Abs	Rel
WACC	0,02%	0,2%
Enterprise Value	302	5,1%

Comment:

Excellent relative difference for the WACC. The one for the Enterprise value is correct. Excellent calibration.

ARKEMA

Risk free rate	3,50%
Beta (Datastream)	-
Unleveraged beta	1,1
Leveraged beta	1,2
Market risk premium	7,50%
Cost of equity (k)	**12,5%**
Beta of the debt	-
Cost of net debt	
Pretax (based on CAPM)	3,18%
Post tax	**2,10%**
WACC (K)	**11,16%**

Perpetuity growth rate	**2,5%**

Sum of discounted FCF	2 066
Terminal Value	3 033
Enterprise Value	**5 099**
Last net debt	658
Equity Value	**4 441**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**576**	**630**	**688**	**722**	**757**	**792**	**829**	**867**
(Corporate tax on EBIT)		(214)	(234)	(245)	(257)	(269)	(282)	(295)
Corporate tax rate		34%	34%	34%	34%	34%	34%	34%
NOPAT		415	454	476	499	523	547	572
D&A		289	291	303	315	328	342	356
(Net CAPEX)		(336)	(332)	(341)	(349)	(358)	(367)	(376)
(DWCR)		27	(45)	(14)	(14)	(15)	(15)	(15)
Free Cash Flow		**396**	**368**	**425**	**451**	**479**	**508**	**537**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**356**	**298**	**309**	**296**	**282**	**269**	**256**

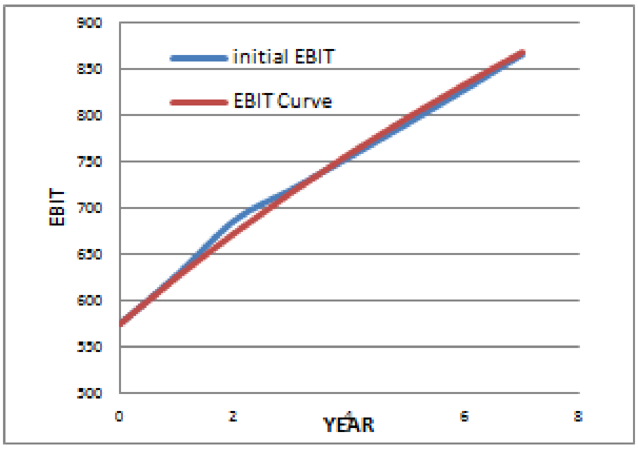
ARKEMA

Parameters

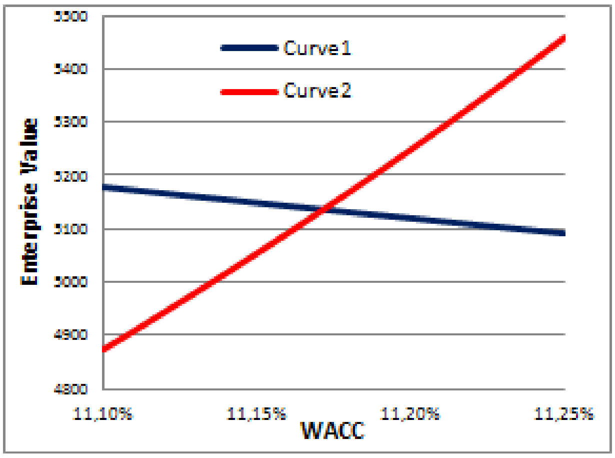
Curve 1	
initial EBIT (EBIT_0)	576
initial growth (g_0)	9,2%
Perpetuity growth (g_inf)	2,5%
Transition Period (T)	24
Tax Rate (tau)	34,0%
DWCR	15

Curve 2	
Net Debt	658
Cost of Equity	12,50%
Cost of Debt (post tax)	2,13%

Calibration



Valuation



Results

	Usual DCF	Continuous DCf
WACC	11,16%	11,17%
Enterprise Value	5099	5134

	Difference	
	Abs	Rel
WACC	0,01%	0,1%
Enterprise Value	35	0,7%

Comment:

Excellent relative difference for the EV and the WACC. Excellent calibration.

M6

Risk free rate	3,50%
Beta (Datastream)	1,02
Unleveraged beta	1,28
Leveraged beta	1,167
Market risk premium	6,50%
Cost of equity (k)	**11,09%**
Beta of the debt	-
Cost of net debt	
Pretax (based on CAPM)	4,80%
Post tax	**3,07%**
WACC (K)	**12,80%**

Sum of discounted FCF	834
Terminal Value	928
Enterprise Value	**1 762**
Last net debt	(377)
Equity Value	**2 139**

Perpetuity growth rate	**3,0%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**258**	**255**	**277**	**284**	**292**	**300**	**309**	**318**
(Corporate tax on EBIT)		(92)	(100)	(103)	(105)	(108)	(112)	(115)
Corporate tax rate		36%	36%	36%	36%	36%	36%	36%
NOPAT		163	177	181	187	192	197	203
D&A		99	102	105	107	110	114	117
(Net CAPEX)		(94)	(96)	(100)	(104)	(109)	(113)	(117)
(DWCR)		1	1	1	1	1	1	2
Free Cash Flow		**169**	**184**	**187**	**191**	**194**	**199**	**205**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**150**	**145**	**131**	**118**	**106**	**97**	**88**

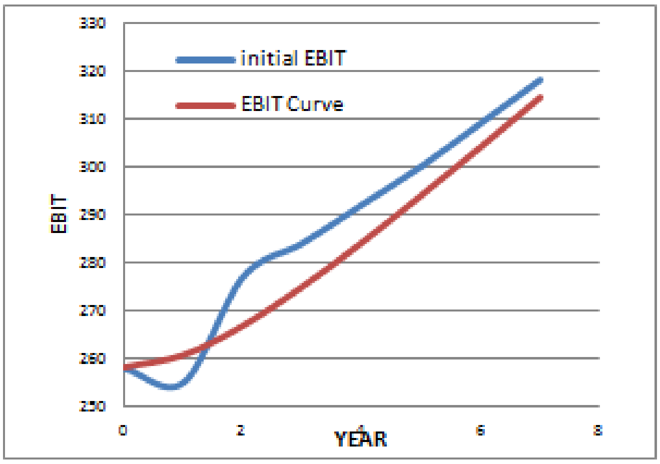
M6

Parameters

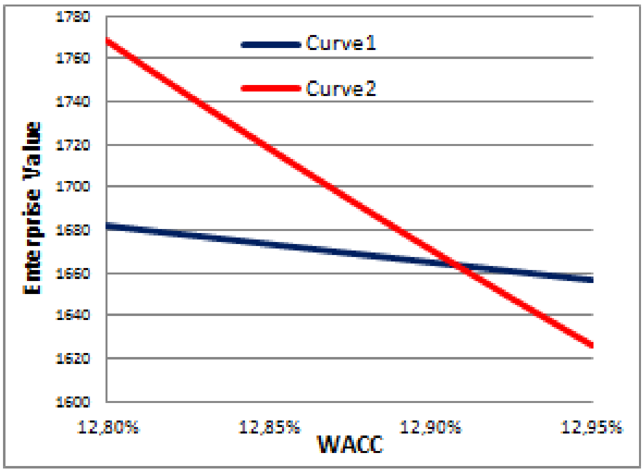
Curve 1	
initial EBIT (EBIT_0)	258
initial growth (g_0)	0,0%
Perpetuity growth (g_inf)	3,0%
Transition Period (T)	7
Tax Rate (tau)	36,1%
DWCR	-2

Curve 2	
Net Debt	-377
Cost of Equity	11,09%
Cost of Debt (post tax)	3,07%

Calibration



Valuation



Results

	Usual DCF	Continuous DCF
WACC	12,80%	12,88%
Enterprise Value	1762	1669

	Difference	
	Abs	Rel
WACC	0,08%	0,6%
Enterprise Value	93	5,3%

Comment:

Excellent relative difference the WACC. The one for the EV is correct. The calibration is quite complicated since the initial EBIT curve is highly volatile

PAJES JAUNES

Risk free rate	2,60%
Beta (Datastream)	1
Unleveraged beta	0,57
Leveraged beta	0,69
Market risk premium	7,50%
Cost of equity (k)	**7,76%**
Beta of the debt	-
Cost of net debt	
Pretax (based on CAPM)	4,10%
Post tax	**2,62%**
WACC (K)	**5,98%**

Perpetuity growth rate	**1,0%**

Sum of discounted FCF	1 595
Terminal Value	3 898
Enterprise Value	**5 493**
Last net debt	1 900
Equity Value	**3 593**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**477**	**477**	**479**	**476**	**476**	**479**	**481**	**483**
(Corporate tax on EBIT)		(172)	(173)	(172)	(172)	(173)	(174)	(174)
Corporate tax rate		36%	36%	36%	36%	36%	36%	36%
NOPAT		305	306	304	304	306	307	309
D&A		18	19	19	19	19	20	20
(Net CAPEX)		(43)	(44)	(44)	(45)	(46)	(46)	(47)
(DWCR)		4	6	3	5	6	8	7
Free Cash Flow		**284**	**287**	**282**	**283**	**285**	**289**	**289**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**268**	**256**	**237**	**224**	**213**	**204**	**192**

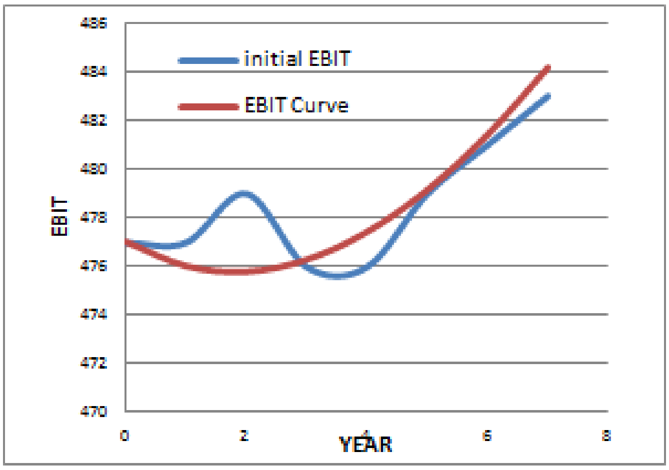
PAGES JAUNES

Parameters

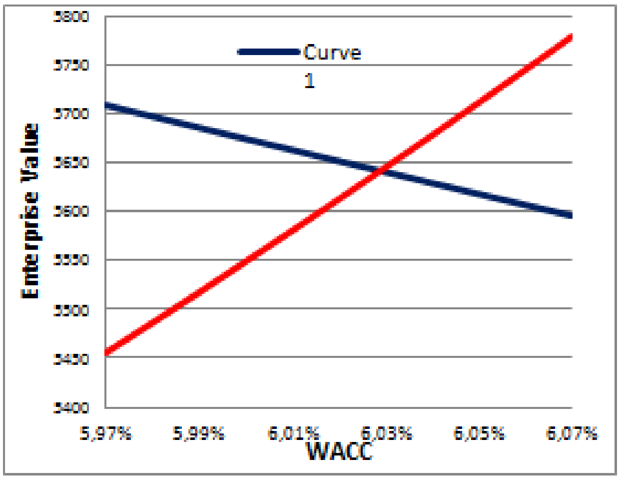
Curve 1	
initial EBIT (EBIT_0)	477
initial growth (g_0)	-0,3%
Perpetuity growth (g_inf)	1,0%
Transition Period (T)	40
Tax Rate (tau)	36,1%
DWCR	8

Curve 2	
Net Debt	1900
Cost of Equity	7,76%
Cost of Debt (post t:	2,62%

Calibration



Valuation



Results

	Usual DCF	Continuou s DCF
WACC	5,98%	6,03%
Enterprise Value	5493	5640

	Difference	
	Abs	Rel
WACC	0,05%	0,8%
Enterprise Valu	147	2,7%

Comment:

Excellent relative difference the WACC and the EV. The calibration is quite complicated since the initial EBIT curve is highly volatile at the beginning.

CARREFOUR

Risk free rate	3,50%
Beta (Datastream)	1,2
Unleveraged beta	0,80
Leveraged beta	1,10
Market risk premium	7,50%
Cost of equity (k)	**11,8%**
Beta of the debt	0,10
Cost of net debt	
Pretax (based on CAPM)	4,25%
Post tax	**2,72%**
WACC (K)	**8,55%**

Sum of discounted FCF	6,4
Terminal Value	16,2
Enterprise Value	**22,6**
Last net debt	8,0
Equity Value	**14,6**

Perpetuity growth rate	**2,0%**

in Bn	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**2,2**	**2,3**	**2,4**	**2,5**	**2,6**	**2,7**	**2,7**	**2,7**
(Corporate tax on EBIT)		(0,8)	(0,9)	(0,9)	(0,9)	(1,0)	(1,0)	(1,0)
Corporate tax rate		36%	36%	36%	36%	36%	36%	36%
NOPAT		1,5	1,5	1,6	1,7	1,7	1,7	1,8
D&A		1,8	1,9	2,0	2,0	2,1	2,1	2,1
(Net CAPEX)		(2,8)	(2,9)	(2,7)	(2,6)	(2,4)	(2,3)	(2,1)
(DWCR)		0,2	0,3	0,3	0,3	0,2	0,2	0,1
Free Cash Flow		**0,7**	**0,8**	**1,2**	**1,4**	**1,6**	**1,7**	**1,9**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**0,62**	**0,71**	**0,94**	**0,98**	**1,08**	**1,05**	**1,04**

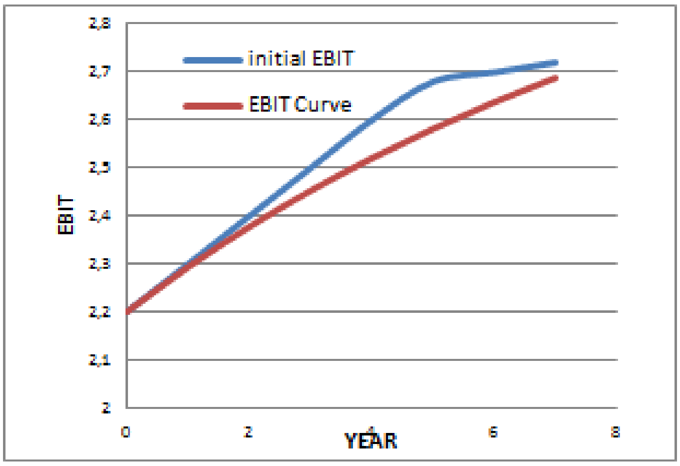
CARREFOUR

Parameters

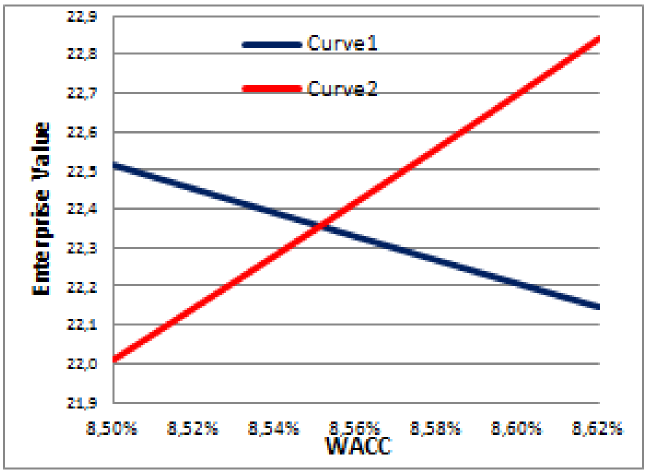
Curve 1	
initial EBIT (EBIT_0)	2,2
initial growth (g_0)	4,5%
Perpetuity growth (g_inf)	1,3%
Transition Period (T)	25
Tax Rate (tau)	36,1%
DWCR	-0,1

Curve 2	
Net Debt	8
Cost of Equity	11,80%
Cost of Debt (post tax)	2,72%

Calibration



Valuation



Results

	Usual DCF	Continuous DCf
WACC	8,55%	8,55%
Enterprise Value	22,6	22,35

	Difference	
	Abs	Rel
WACC	0,00%	0,0%
Enterprise Value	0,30	1,3%

Comment:

Excellent relative difference the WACC and the EV. The calibration is good, a little bit under the initial curve.

CLUB MED

Risk free rate	3,50%
Beta (Datastream)	-
Unleveraged beta	-
Leveraged beta	1,20
Market risk premium	7,50%
Cost of equity (k)	**12,5%**
Beta of the debt	0,4
Cost of net debt	
Pretax (based on CAPM)	6,50%
Post tax	**4,15%**
WACC (K)	**10,31%**

Perpetuity growth rate	**2,0%**

Sum of discounted FCF	287
Terminal Value	344
Enterprise Value	**631**
Last net debt	165
Equity Value	**466**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**61**	**65**	**70**	**73**	**75**	**77**	**79**	**81**
(Corporate tax on EBIT)		(23)	(25)	(26)	(27)	(28)	(29)	(29)
Corporate tax rate		36%	36%	36%	36%	36%	36%	36%
NOPAT		42	45	47	48	49	50	52
D&A		66	71	74	76	78	80	82
(Net CAPEX)		(53)	(55)	(60)	(65)	(69)	(74)	(79)
(DWCR)		1	2	2	2	2	1	1
Free Cash Flow		**56**	**63**	**63**	**61**	**60**	**57**	**56**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**50**	**52**	**47**	**41**	**37**	**32**	**28**

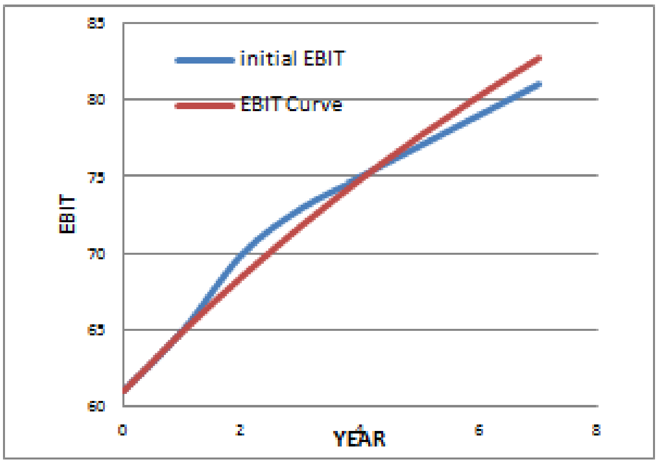
CLUB MED

Parameters

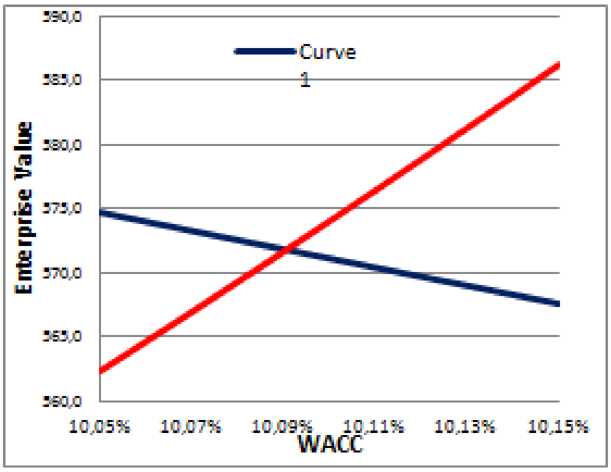
Curve 1	
initial EBIT (EBIT_0)	61
initial growth (g_0)	6,7%
Perpetuity growth (g_inf)	2,0%
Transition Period (T)	25
Tax Rate (tau)	36,1%
DWCR	-2

Curve 2	
Net Debt	165
Cost of Equity	12,50%
Cost of Debt (post t	4,15%

Calibration



Valuation



Results

	Usual DCF	Continuous DCf
WACC	10,31%	10,09%
Enterprise Value	630,9	571,8

	Difference	
	Abs	Rel
WACC	0,22%	2,1%
Enterprise Valu	59,1	9,4%

Comment:

Good relative difference the WACC. The one for the EV is only acceptable.
The calibration is good, a little bit under the initial curve.

SANOFI

Risk free rate	3,50%
Beta (Datastream)	0,8
Unleveraged beta	0,8
Leveraged beta	0,8
Market risk premium	7,50%
Cost of equity (k)	**9,5%**
Beta of the debt	-
Cost of net debt	
Pretax (based on CAPM)	4,38%
Post tax	**2,80%**
WACC (K)	**9,40%**

Sum of discounted FCF	36 349
Terminal Value	64 895
Enterprise Value	**101 244**
Last net debt	1 577
Equity Value	**99 667**

Perpetuity growth rate	**2,5%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**11 481**	**11 038**	**11 621**	**11 955**	**12 287**	**12 617**	**12 945**	**13 268**
(Corporate tax on EBIT)		(3 985)	(4 195)	(4 316)	(4 436)	(4 555)	(4 673)	(4 790)
Corporate tax rate		36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%
NOPAT		7 053	7 426	7 639	7 851	8 062	8 272	8 478
D&A		1 541	1 569	1 615	1 659	1 704	1 748	1 792
(Net CAPEX)		(1 709)	(1 820)	(1 814)	(1 808)	(1 803)	(1 797)	(1 792)
(DWCR)		(237)	(295)	(294)	(293)	(291)	(288)	(285)
Free Cash Flow		**6 648**	**6 880**	**7 146**	**7 409**	**7 672**	**7 935**	**8 193**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**6 077**	**5 748**	**5 458**	**5 173**	**4 896**	**4 628**	**4 369**

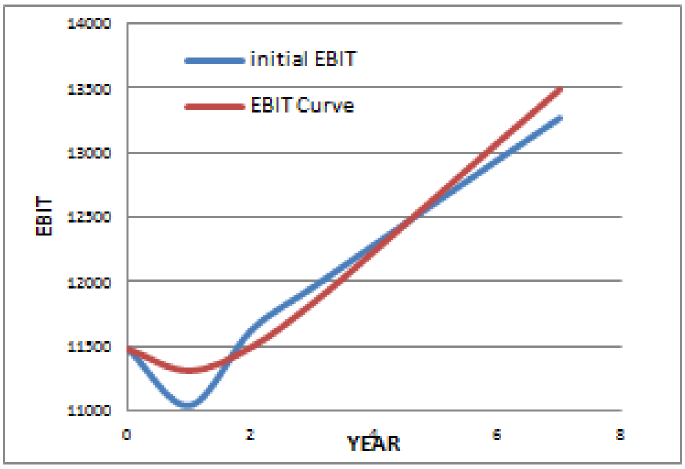
SANOFI

Parameters

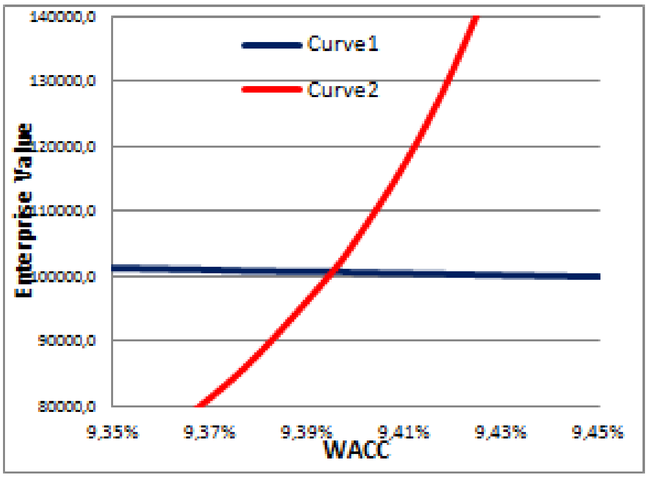
Curve 1	
initial EBIT (EBIT_0)	11481
initial growth (g_0)	-3,9%
Perpetuity growth (g_inf)	2,5%
Transition Period (T)	6
Tax Rate (tau)	36,1%
DWCR	290

Curve 2	
Net Debt	1577
Cost of Equity	9,50%
Cost of Debt (post tax)	2,80%

Calibration



Valuation



Results

	Usual DCF	Continuous DCF
WACC	9,40%	9,395%
Enterprise Value	101244	106035

	Difference Abs	Difference Rel
WACC	0,00%	0,1%
Enterprise Value	4791,4	4,7%

Comment:

Excellent relative difference the WACC. The one for the EV is good (under 5%).

BIOMERIEUX

Risk free rate	3,50%
Beta (Datastream)	0,8
Unleveraged beta	0,806
Leveraged beta	0,802
Market risk premium	7,50%
Cost of equity (k)	**9,5%**
Beta of the debt	0,1
Cost of net debt	
Pretax (based on CAPM)	5,0%
Post tax	**3,30%**
WACC (K)	**9,58%**

Sum of discounted FCF	834
Terminal Value	1 490
Enterprise Value	**2 324**
Last net debt	(25)
Equity Value	**2 349**

Perpetuity growth rate	**2,5%**

In M€	2011	2012	2013	2014	2015	2016	2017	2018
EBIT	**265**	**302**	**331**	**352**	**371**	**387**	**401**	**411**
(Corporate tax on EBIT)		(103)	(113)	(120)	(126)	(132)	(136)	(140)
Corporate tax rate		34%	34%	34%	34%	34%	34%	34%
NOPAT		199	218	232	245	255	265	271
D&A		105	122	130	137	143	148	151
(Net CAPEX)		(148)	(158)	(170)	(181)	(193)	(205)	(216)
(DWCR)		(31)	(24)	(23)	(20)	(18)	(14)	(11)
Free Cash Flow		**125**	**158**	**169**	**181**	**187**	**194**	**195**
Discount period		1	2	3	4	5	6	7
Discounted FCF		**114**	**132**	**129**	**125**	**119**	**112**	**103**

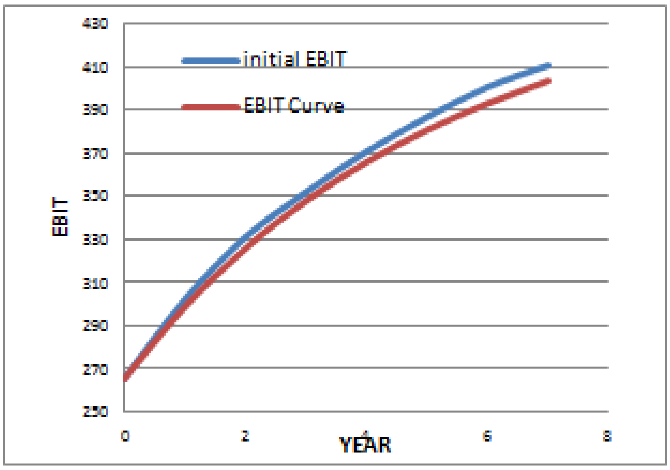
BIOMERIEUX

Parameters

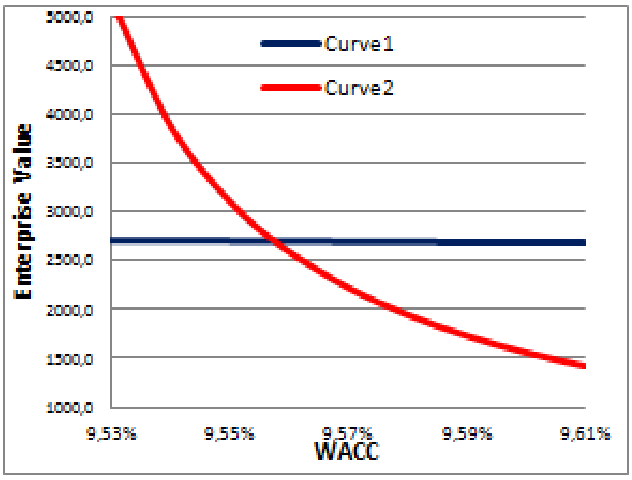
Curve 1	
initial EBIT (EBIT_0)	265
initial growth (g_0)	14,0%
Perpetuity growth (g_inf)	2,5%
Transition Period (T)	13
Tax Rate (tau)	34,0%
DWCR	14

Curve 2	
Net Debt	-25
Cost of Equity	9,5%
Cost of Debt (post tax)	3,3%

Calibration



Valuation



Results

	Usual DCF	Continuous DCf
WACC	9,58%	9,557%
Enterprise Value	2324	2680

	Difference	
	Abs	Rel
WACC	0,02%	0,2%
Enterprise Value	356,2	15,3%

Comment:

Excellent relative difference the WACC. The one for the EV is quite bad. This result is due to the fact that in the usual DCF the D&A and the CAPEX do not compensate each other. Good calibration.

SCILAB CODE

```
function [H]=DCF(ebit,gzero, ginf, T , k, tau);
 lambda=4*log(2)/T;
 k=log(1+k);
 gzero=log(1+gzero);
 ginf=log(1+ginf);
 EV=(1-tau)*ebit*(1/(k-ginf)+1/(k+lambda-gzero)-1/(k+lambda-ginf));
 EVbis=(1-tau)*ebit*(1/(k-ginf));
 H=[EV,EVbis]
endfunction


function u=courbe(ebit,gzero, ginf, T , k, tau)
 u = [0.081:0.001:0.123];
 v=[];
 Tableau=[];
 for i=1:length(u)
  Tableau=DCF(ebit,gzero, ginf, T , u(i), tau)
  v(i)=Tableau(1)
 end;
  plot2d(u,v);
endfunction;

function u=courbe2(ebit,gzero, ginf, T , k, tau, deltaWC)
 u = [0.118:0.001:0.123];
 v=[];
 Tableau=[];
 for i=1:length(u)
  Tableau=DCF(ebit,gzero, ginf, T , u(i), tau)
  v(i)=Tableau(1)-deltaWC/u(i);

 end;
  plot2d(u,v);
endfunction;

function u=growth(gzero, ginf, T )
 u = [0.0:0.1:10];
 lambda=4*log(2)/T;
 v=[];
 l=[];
 Tableau=[];
 for i=1:length(u)
  v(i)=exp((gzero-lambda)*u(i))+(1-exp(-lambda*u(i)))*exp(ginf*u(i))
  l(i)=exp(gzero*u(i));
  k(i)=exp(ginf*u(i));
 end;
  plot2d(u,v);
  plot2d(u,l);
  plot2d(u,k);
endfunction;
```

```
function u=growth2(gzero, ginf, T )
 u = [0.08:0.1:15];
 lambda=4*log(2)/T;
 v=[];
 l=[];
 Tableau=[];
 for i=1:length(u)
  v(i)=log(exp((gzero-lambda)*u(i))+(1-exp(-lambda*u(i)))*exp(ginf*u(i)))/u(i);
  l(i)=gzero;
  k(i)=ginf;
 end;
  plot2d(u,v);
  plot2d(u,l);
  plot2d(u,k);
endfunction;

function u=EBIT(Ebit0, gzero, ginf, T )
 u = [0.1:0.1:10];
 lambda=4*log(2)/T;
 v=[];
 l=[];
 Tableau=[];
 for i=1:length(u)
  v(i)=Ebit0*(exp((gzero-lambda)*u(i))+(1-exp(-lambda*u(i)))*exp(ginf*u(i)));
 end;
  plot2d(u,v);
endfunction;


function v=EV(ND,kd,ke,tau)

u=[0.118:0.0001:0.13]
v=[];
for i=1:length(u)
v(i)= -ND*(kd*(1-tau)-ke)/(exp(u(i))-1-ke);
end

plot2d(u,v);

endfunction

//////////////fsolve-example///////////////////
function [h1]=resoudre()

function [y]=fcta(x)
y=2*x^3-30*x^2-3*x+200,
endfunction
```

```
h=[-3:0.1:15];//xbasc();
plot2d(h,fcta(h));
h1=fsolve(-1,fcta)
endfunction

//////////////Solving the problem///////////////////

function [v]=EVandWacc(ND,kd,ke,tau,ebit,gzero, ginf, T, deltaWC)

function [v2]=FromWacc(v1)//WACC equation
v2=-ND*(kd*(1-tau)-ke)/(exp(v1)-1-ke);
endfunction

function [u2]=FromDCF(u1)//DCF equation
Tableau=DCF(ebit,gzero, ginf, T , u1, tau)-deltaWC/u1;
u2=Tableau(1);
endfunction

function [w2]=Diff(w1)//Differnce
w2=FromDCF(w1)-FromWacc(w1);
endfunction

//Solving the System with the Newton Raphson method
xxx=fsolve(0.12,Diff);
v=[floor(xxx*10000)/10000,floor(FromWacc(xxx))];
a=xxx-0.0001;
b=xxx+0.0001;

u=[a:0.00001:b];
//plot2d(u,FromWacc(u));

res=[];
resbis=[];
for i=1:length(u)
res(i)=FromDCF(u(i));
resbis(i)=FromWacc(u(i));
end
plot2d(u,res,style=5, leg="EV from DCF");
plot2d(u,resbis,leg="EV from Wacc");

endfunction
```